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13. ABSTRACT

The motion of a satellite with respect to a fixed system of coordinates in space has been determined. Formulas are derived which determine the aspect of the satellite axis, and the aspect of a vector perpendicular to the satellite axis. The telemetered data consisted of solar angle measurements in terms of voltage from six sun sensors along the pitch, yaw, and roll axis, and magnetic field measurements from three mutually perpendicular magnetometers.

The 7094 computer programs to determine the aspects along with the resulting plots of the desired angles as a function of flight time for different revolutions are exhibited. These plots showed that the satellite was not stable as expected, but a stabilizing trend was noticeable as flight time increased.

KEYWORDS: Solar Sensor, Aspect Angle, Magnetometer, Vertistat

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ASPECT OF THE AXIS AND OF A VECTOR PERPENDICULAR TO THE AXIS OF THE SATELLITE OV1-5

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APPENDIX

ABSTRACT

The motion of a satellite with respect to a fixed system of coordinates in space has been determined. Formulas are derived which determine the aspect of the satellite axis, and the aspect of a vector perpendicular to the satellite axis. The telemetered data consisted of solar angle measurements in terms of voltage from six sun sensors along the pitch, yaw, and roll axis, and magnetic field measurements from three mutually perpendicular magnetometers.

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NOMENCLATURE

Symbol	
e_r, e_Θ, e_Φ	system of orthonormal base vectors on OV1-5 with e_r along the nose axis, e_Θ in the direction of sun sensor E, and $e_\Phi = e_\Theta \times e_r$
α_1, α_2	the angles determined by output voltage #1 and output voltage #2 respectively for sun sensor A
β_1, β_2	the angles determined by output voltage #1 and output voltage #2 respectively for sun sensor B
γ_1, γ_2	the angles determined by output voltage #1 and output voltage #2 respectively for sun sensor C
δ_1, δ_2	the angles determined by output voltage #1 and output voltage #2 respectively for sun sensor D
ξ_1, ξ_2	the angles determined by output voltage #1 and output voltage #2 respectively for sun sensor E
f_1, f_2	the angles determined by output voltage #1 and output voltage #2 respectively for sun sensor F
i, j, k	system of orthonormal base vectors in a right handed fixed system with i and j in the equatorial plane and i parallel to the vernal equinox
$\hat{S}$	a unit vector parallel to the sun's rays to the satellite, but in opposite sense
$\hat{M}$	a unit vector parallel to the earth's magnetic field
Θ_s	declination of the sun
Φ_s	apparent right ascension of the sun
Θ_H	angle between the equatorial plane and the magnetic field
Φ_H	azimuth of the magnetic field in the fixed system i, j, k

Θ	angle between the nose axis of the satellite and the equatorial plane
Φ	azimuth of the nose axis with respect to the vernal equinox
Θ_E, Φ_E	latitude and longitude of the satellite from the ephemeris
ω	the angular velocity of rotation of the earth on its axis with respect to the vernal equinox
$\hat{N}_1$, $\hat{N}_2$	orthogonal unit vectors defining a plane where e_Φ rotates
ρ	the angle between N_1 and e_Φ
Ψ_H	the angle between the magnetic field vector and the equatorial plane of the earth
λ_H	the azimuth of the magnetic field vector with respect to the Greenwich Meridian Plane
$\ddot{i}$, $\dot{j}$, $\ddot{k}$	geocentric system of base unit vectors
F	total magnetic field
R_E	vector from the earth's center to the satellite in the rotating system
β_s	angle between the axis of the satellite and the sun vector
β_H	angle between the axis of the satellite and the magnetic field vector
$\hat{U}''$	unit vector from the center of the earth to the satellite
γ_s	angle between e_Φ and the sun vector

ILLUSTRATIONS

INTRODUCTION

To properly analyze the data of certain satellite detectors one must know the angle between the axis of the detector and a specified vector in space. The goal in this report is to determine the angle between a detector perpendicular to the satellite axis and a vector from this detector to the earth's center. This was done by considering the angle between a vector from the center of the earth (which we will call $\hat{U}''$) to the satellite and a vector (which we will call e_ϕ) perpendicular to the axis of the satellite and in the opposite direction from the detector.

In order to obtain this angle it is necessary to first determine the motion of the satellite about the pitch, yaw, and roll axes. In the early portions of the flight of OV1-5, motion about each of these axes was active indicating that the satellite was quite unstable. However at revolution 957 the motion became approximately 1.5 turns on the pitch axis and roll axis with rotation of approximately 45 degrees on the yaw axis. At revolution 1360, complete turns ceased on each of the axes and the satellite seemed to be quite stable. It should be noted that a stabilizing trend was apparent after revolution 957 but the degree of stabilization could

only be determined by analysis of the particular revolution.

The aspect of the axis of the satellite was also necessary as input data for the analysis of the aspect of the vector perpendicular to the axis, that is the angle between e_ϕ and $\hat{U}''$ as a function of flight time.

CHAPTER I

DESCRIPTION OF OV1-5 ASPECT SYSTEM

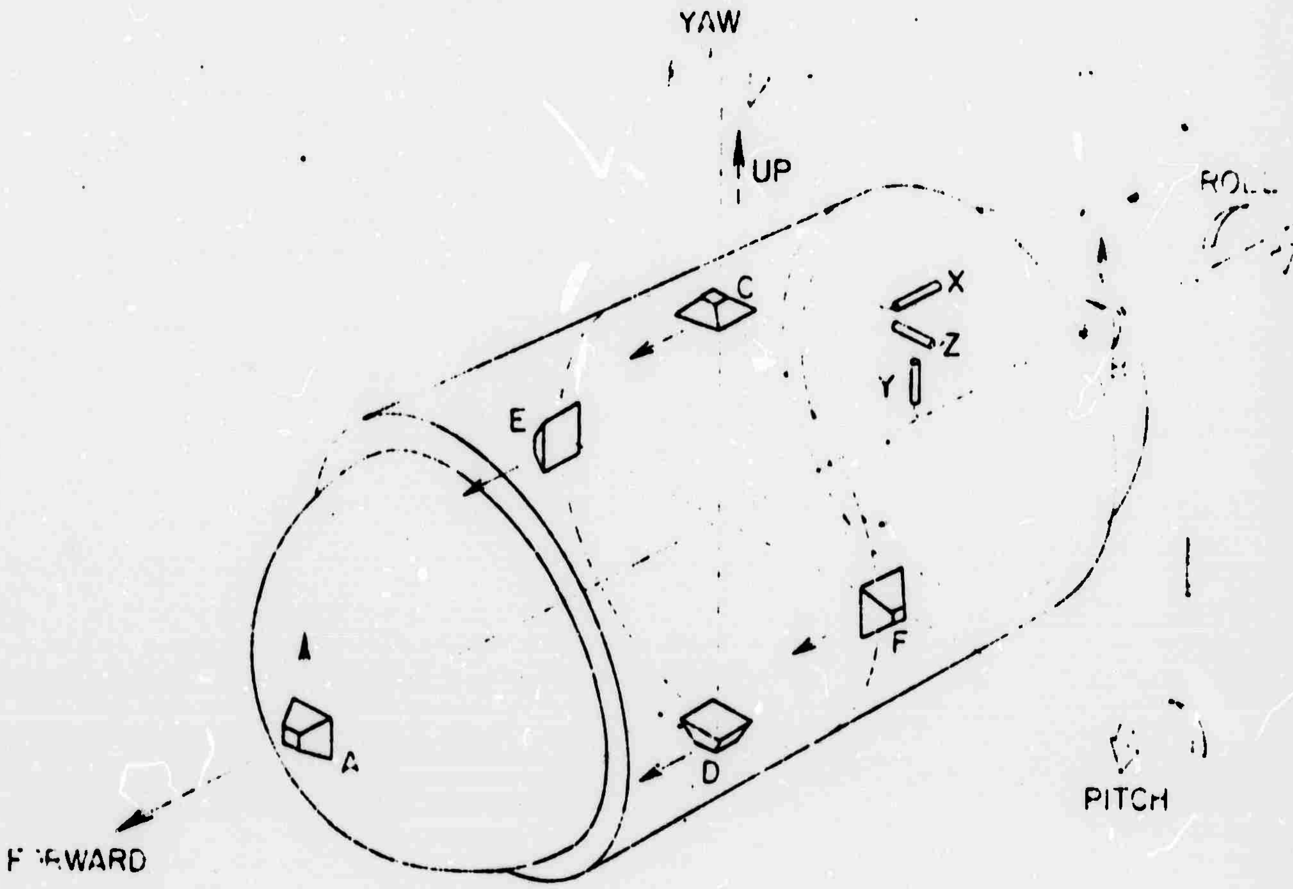
The responsibility for designing a suitable spacecraft aspect detection system was given to American Science and Engineering, Inc. This system consisted of six solar aspect sensors and three magnetometers. For the sun sensors, calibration consists of determining the two output voltages for each sensor which result from the light source of the sun. The requirement for the sun sensors are a clear 45° conical field of view with axis alignment consistent with the magnetometers. Only one sun sensor is recording at a given time and that sensor is determined by the appropriate recorded signature voltage.

A. Position of Sun Sensors and Magnetometers

The OV1-5 aspect system sensor locations and orientation along with the calibration information is shown in the following pages.* The pitch magnetometer X determines the component of the magnetic field sensed along the pitch axis; the yaw magnetometer Z determines the component of the magnetic field sensed along the yaw axis and the roll magnetometer Y determines the component of the magnetic field sensed along the roll axis. Solar aspect output #1 is used to determine the angle between the sun and the satellite in the plane of the reference arrow marked on the appropriate sun sensor. Solar aspect output #2 is used to determine the angle between the sun and the satellite in the plane perpendicular to the arrow marked on the appropriate sun sensor.

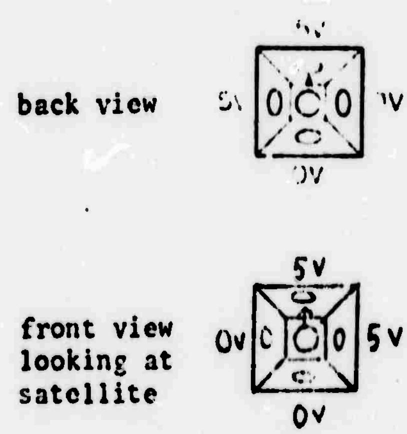
*This information was provided by American Science and Engineering, Inc., Cambridge, Massachusetts.

OV1-5 ASPECT SYSTEM SENSOR LOCATIONS AND ORIENTATION



OUTPUT SIGNAL SENSE

SOLAR ASPECT SYSTEM



MAGNETOMETERS

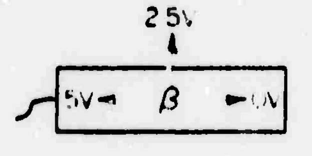


Figure 1

SOLAR ASPECT SYSTE[illegible]

	SENSOR A		SENSOR B		SE[illegible]
Signature Voltage =	4.69		3.90		
	ASPECT OUTPUT		ASPECT OUTPUT		ASPE[illegible]
	#1	#2	#1	#2	#1
ANGLE					
45	4.27	4.23	4.24	4.24	4.24
40	3.96	3.90	3.86	3.87	3.87
30	3.33	3.30	3.24	3.31	3.23
20	2.92	2.87	2.82	2.87	2.86
10	2.57	2.52	2.47	2.50	2.52
0	2.26	2.21	2.16	2.18	2.21
10	1.95	1.89	1.83	1.86	1.90
20	1.61	1.54	1.48	1.51	1.56
30	1.21	1.11	1.04	1.09	1.14
40	0.63	0.48	0.35	0.50	0.53
45	0.29	0.23	0.26	0.24	0.26

SOLAR ASPECT SYSTEM CALIBRATION

A	SENSOR B 3.90		SENSOR C 3.19		SENSOR D 2.42		SENS
UTPUT	ASPECT OUTPUT		ASPECT OUTPUT		ASPECT OUTPUT		ASPECT
#2	#1	#2	#1	#2	#1	#2	#1
4.23	4.24	4.24	4.24	4.28	4.25	4.23	4.35
3.90	3.86	3.97	3.87	4.04	3.93	3.37	4.26
3.30	3.24	3.31	3.23	3.35	3.28	3.27	3.51
2.87	2.82	2.87	2.86	2.88	2.80	2.86	2.99
2.52	2.47	2.50	2.52	2.51	2.51	2.51	2.57
2.21	2.16	2.18	2.21	2.19	2.20	2.19	2.19
1.89	1.83	1.86	1.90	1.87	1.89	1.85	1.81
1.54	1.43	1.51	1.56	1.54	1.56	1.51	1.42
1.11	1.04	1.09	1.14	1.16	1.15	1.07	0.97
0.48	0.35	0.50	0.53	0.65	0.54	0.45	0.42
0.23	0.26	0.24	0.26	0.30	0.28	0.24	0.19

…M CALIBRATION

…NSOR C 3.19	SENSOR D 2.42		SENSOR E .86V		SENSOR F 1.57	
…CT OUTPUT	ASPECT OUTPUT		ASPECT OUTPUT		ASPECT OUTPUT	
#2	#1	#2	#1	#2	#1	#2
4.28	4.25	4.23	4.35	0.24	4.27	4.26
4.04	3.93	3.87	4.26	0.49	3.94	3.98
3.35	3.28	3.27	3.51	1.07	3.31	3.33
2.88	2.80	2.86	2.99	1.52	2.89	2.88
2.51	2.51	2.51	2.57	1.88	2.54	2.52
2.19	2.20	2.19	2.19	2.23	2.21	2.20
1.87	1.89	1.85	1.81	2.59	1.90	1.87
1.54	1.56	1.51	1.42	2.97	1.54	1.53
1.10	1.15	1.07	0.97	3.43	1.10	1.12
0.65	0.54	0.45	0.42	4.05	0.45	0.51
0.30	0.28	0.24	0.19	4.30	0.28	0.23

MAGNETOMETER CALIBRATION INFORMATION

Field in Milligauss	X	Y	Z
600	4.81	4.80	4.81
550	4.63	4.62	4.65
500	4.45	4.43	4.47
450	4.25	4.23	4.25
400	4.04	4.01	4.05
350	3.83	3.79	3.81
300	3.61	3.57	3.57
250	3.39	3.35	3.34
200	3.18	3.14	3.13
150	2.98	2.94	2.93
100	2.78	2.76	2.71
50	2.59	2.58	2.57
0	2.41	2.41	2.40
-50	2.22	2.24	2.23
-100	2.03	2.06	2.05
-150	1.84	1.87	1.86
-200	1.63	1.68	1.67
-250	1.42	1.47	1.43
-300	1.21	1.26	1.22
-350	.99	1.03	.98
-400	.78	.82	.75
-450	.58	.61	.53
-500	.38	.40	.32
-550	.19	.21	+.14
-600	+.02	+.03	-.03

B. <u>Determination of Sun Angles from Calibration Curves</u>

Using the calibration information provided, a least squares cubic fit was made to each of the two output voltages for all six sun sensors. The method was as follows:

$$\Theta = A + BV + CV^2 + DV^3$$

where V equals the output voltage, Θ equals the sun-satellite angle, and A, B, C, D are the constants to be determined.

Let

$$I_n = \sum_{k=1}^{n} [A + BV_k + CV_k{}^2 + DV_k{}^3 = \Theta_k]^2$$

$$\Rightarrow \frac{\partial In}{\partial A} = \frac{\partial In}{\partial B} = \frac{\partial In}{\partial C} = \frac{\partial In}{\partial D} = 0$$

$$\begin{aligned} \Rightarrow An + B\sum V_k + C\sum V_k{}^2 + D\sum V_k{}^3 &= \sum \Theta_k \\ A\sum V_k + B\sum V_k{}^2 + C\sum V_k{}^3 + D\sum V_k{}^4 &= \sum V_k \Theta_k \\ A\sum V_k{}^2 + B\sum V_k{}^3 + C\sum V_k{}^4 + D\sum V_k{}^5 &= \sum V_k{}^2 \Theta_k \\ A\sum V_k{}^3 + B\sum V_k{}^4 + C\sum V_k{}^5 + D\sum V_k{}^6 &= \sum V_k{}^3 \Theta_k \end{aligned}$$

Using Cramer's rule we can now find the desired constants.

$$\Delta = \begin{vmatrix} n & \sum V_k & \sum V_k{}^2 & \sum V_k{}^3 \\ \sum V_k & \sum V_k{}^2 & \sum V_k{}^3 & \sum V_k{}^4 \\ \sum V_k{}^2 & \sum V_k{}^3 & \sum V_k{}^4 & \sum V_k{}^5 \\ \sum V_k{}^3 & \sum V_k{}^4 & \sum V_k{}^5 & \sum V_k{}^6 \end{vmatrix}$$

$$A = \frac{\begin{vmatrix} \sum\Theta_k & \sum v_k & \sum v_k{}^2 & \sum v_k{}^3 \\ \sum v_k\Theta_k & \sum v_k{}^2 & \sum v_k{}^3 & \sum v_k{}^4 \\ \sum v_k{}^2\Theta_k & \sum v_k{}^3 & \sum v_k{}^4 & \sum v_k{}^5 \\ \sum v_k{}^3\Theta_k & \sum v_k{}^4 & \sum v_k{}^5 & \sum v_k{}^6 \end{vmatrix}}{\Delta}$$

$$B = \frac{\begin{vmatrix} n & \sum\Theta_k & \sum v_k{}^2 & \sum v_k{}^3 \\ \sum v_k & \sum v_k\Theta_k & \sum v_k{}^3 & \sum v_k{}^4 \\ \sum v_k{}^2 & \sum v_k{}^2\Theta_k & \sum v_k{}^4 & \sum v_k{}^5 \\ \sum v_k{}^3 & \sum v_k{}^3\Theta_k & \sum v_k{}^5 & \sum v_k{}^6 \end{vmatrix}}{\Delta}$$

$$C = \frac{\begin{vmatrix} n & \sum v_k & \sum\Theta_k & \sum v_k{}^3 \\ \sum v_k & \sum v_k{}^2 & \sum v_k\Theta_k & \sum v_k{}^4 \\ \sum v_k{}^2 & \sum v_k{}^3 & \sum v_k{}^2\Theta_k & \sum v_k{}^5 \\ \sum v_k{}^3 & \sum v_k{}^4 & \sum v_k{}^3\Theta_k & \sum v_k{}^6 \end{vmatrix}}{\Delta}$$

$$D = \frac{\begin{vmatrix} n & \sum v_k & \sum v_k{}^2 & \sum\Theta_k \\ \sum v_k & \sum v_k{}^2 & \sum v_k{}^3 & \sum v_k\Theta_k \\ \sum v_k{}^2 & \sum v_k{}^3 & \sum v_k{}^4 & \sum v_k{}^2\Theta_k \\ \sum v_k{}^3 & \sum v_k{}^4 & \sum v_k{}^5 & \sum v_k{}^3\Theta_k \end{vmatrix}}{\Delta}$$

The plots of the different curves for each sensor and output voltage can be found in the appendix.

C. Determination of the Magnetic Field from the Calibration Curves

A straight line fit was made for each magnetometer using the appropriate calibration information. The points and equations used are shown in the appendix along with the linear fits to the calibration curves. It should be noted that the X magnetometer has its direction coincident with the positive e_r axis in the previous discussion. The Y magnetometer is coincident with $-e_\phi$, and the Z magnetometer is coincident with $-e_\Theta$.

CHAPTER II

DETERMINATION OF THE ANGLES BETWEEN THE SUN VECTOR AND THE AXES OF THE SATELLITE

A. Theoretical Description

For sensor A with signature voltage 4.69^V, we have the following set-up:

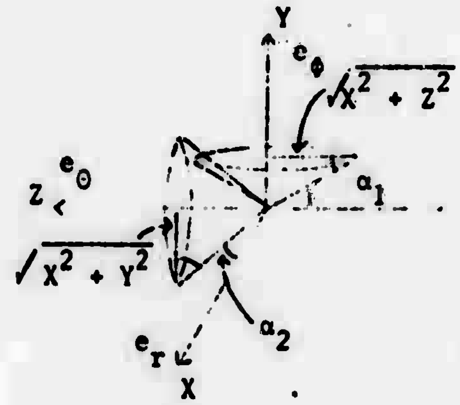


Figure 2

α_1 is the angle determined by output voltage #1 and α_2 is the angle determined by output voltage #2. The sun vector is determined by the intersection of the cones produced by α_1 and α_2; and we will now determine this sun vector. However, it should be noted that although the cones may intersect in two places, the ambiguity is resolved since we know which sensor is reading and therefore on which side the sun lies.

$$\frac{\sqrt{X^2 + Z^2}}{Y} = \cot\alpha_1 \tag{1}$$

$$\frac{\sqrt{X^2 + Y^2}}{Z} = \cot\alpha_2 \tag{2}$$

$$\Rightarrow X^2 + Z^2 = Y^2 \cot^2\alpha_1$$
$$X^2 + Y^2 = Z^2 \cot^2\alpha_2$$

Subtracting, we get

$$Z^2 - Y^2 = Y^2 \cot^2\alpha_1 - Z^2 \cot^2\alpha_2$$
$$Z^2 \csc^2\alpha_2 = y^2 \csc^2\alpha_1$$
$$Z = \pm y \frac{\sin\alpha_2}{\sin\alpha_1}$$

From (1)

$$X^2 = Y^2 \left[\frac{\cos^2\alpha_1}{\sin^2\alpha_1} - \frac{\sin^2\alpha_2}{\sin^2\alpha_1}\right] = Y^2\left[\frac{\cos^2\alpha_1 - \sin^2\alpha_2}{\sin^2\alpha_1}\right]$$

$$\Rightarrow X = \pm \frac{Y \sqrt{\cos^2\alpha_1 - \sin^2\alpha_2}}{\sin\alpha_1}$$

$$Y = Y \frac{\sin\alpha_1}{\sin\alpha_1}$$

$$Z = \pm Y \frac{\sin\alpha_2}{\sin\alpha_1}$$

Therefore any vector R from the satellite to the given light source can be expressed as:

$$R = e_r \left[\pm y \frac{\sqrt{\cos^2\alpha_1 - \sin^2\alpha_2}}{\sin\alpha_1}\right] + e_\Theta \left[\pm y \frac{\sin\alpha_2}{\sin\alpha_1}\right] + e_\phi \, y \frac{\sin\alpha_1}{\sin\alpha_1}$$

Normalizing this vector and expressing it in terms of the sun vector $\hat{S}$ we get:

$$\hat{S} = \pm e_r \sqrt{\cos^2\alpha_1 - \sin^2\alpha_2} \pm e_\Theta \sin\alpha_2 + e_\phi \sin\alpha_1 \tag{3}$$

1. if $\alpha_1 > 0$, $\alpha_2 > 0$

$$\hat{S} = e_r \sqrt{\cos^2\alpha_1 - \sin^2\alpha_2} - e_\Theta \sin|\alpha_2| + e_\phi \sin\alpha_1$$

$\hat{S}$ is in the octant e_r, $-e_\Theta$, e_ϕ

2. if $\alpha_1 > 0$, $\alpha_2 < 0$

$$\hat{S} = e_r \sqrt{\cos^2\alpha_1 - \sin^2\alpha_2} - e_\Theta \sin\alpha_2 + e_\phi \sin\alpha_1$$

$\hat{S}$ is in the octant e_r, e_Θ, e_ϕ

3. if $\alpha_1 < 0$, $\alpha_2 > 0$

$$\hat{S} = e_r \sqrt{\cos^2\alpha_1 - \sin^2\alpha_2} - e_\Theta \sin|\alpha_2| - e_\phi \sin|\alpha_1|$$

$\hat{S}$ is the octant e_r, $-e_\Theta$, $-e_\phi$

4. if $\alpha_1 < 0$, $\alpha_2 < 0$

$$\hat{S} = e_r \sqrt{\cos^2\alpha_1 - \sin^2\alpha_2} - e_\Theta \sin\alpha_2 - e_\phi \sin|\alpha_1|$$

$\hat{S}$ is the octant e_r, e_ϕ, $-e_\phi$

For sun sensor B with signature voltage 3.90^V and output voltages β_1 and β_2 corresponding to α_1 and α_2, we find

1. if $\beta_1 > 0$, $\beta_2 > 0$

$$\hat{S} = -e_r \sqrt{\cos^2\beta_1 - \sin^2\beta_2} + e_\Theta \sin\beta_2 + e_\phi \sin\beta_1$$

$\hat{S}$ is in the octant $-e_r$, e_Θ, e_ϕ

2. if $\beta_1 > 0$, $\beta_2 < 0$

$$\hat{S} = -e_r \sqrt{\cos^2\beta_1 - \sin^2\beta_2} + e_\Theta \sin\beta_2 + e_\phi \sin\beta_1$$

$\hat{S}$ is in the octant $-e_r$, $-e_\Theta$, e_ϕ

3. if $\beta_1 < 0$, $\beta_2 > 0$

$$\hat{S} = -e_r \sqrt{\cos^2\beta_1 - \sin^2\beta_2} + e_\Theta \sin\beta_2 + e_\phi \sin\beta_1$$

$\hat{S}$ is in the octant $-e_r$, e_Θ, $-e_\phi$

4. if $\beta_1 < 0$, $\beta_2 < 0$

$$\hat{S} = -e_r \sqrt{\cos^2\beta_1 - \sin^2\beta_2} + e_\Theta \sin\beta_2 + e_\phi \sin\beta_1$$

$\hat{S}$ is in the octant. $-e_r$, $-e_\Theta$, $-e_\phi$

Continuing with this same approach, we will now find $\dot{S}$ for sensors C and D. For sensor C with signature voltage 3.19^V we have the following figure:

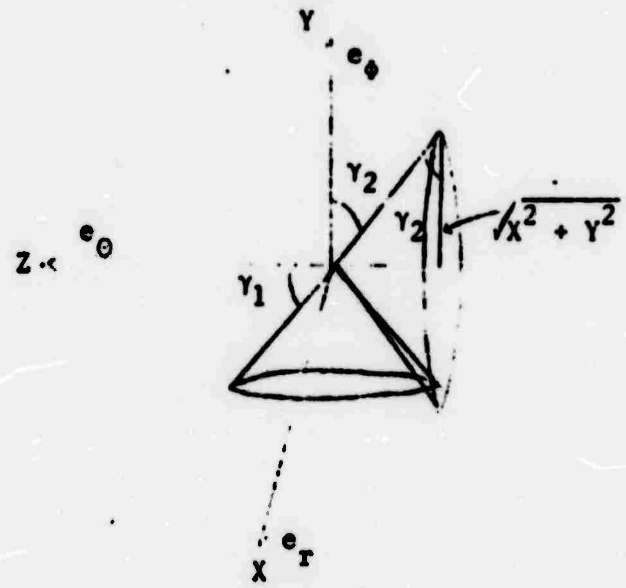


Figure 3

It should be noted that while sensor A faces along positive e_r, sensor C faces along positive e_ϕ. γ_1 and γ_2 are the angles produced by the output voltages where γ_1 corresponds to output voltage #1.

$$X^2 + Y^2 = Z^2 \cot^2\gamma_2 \tag{4}$$

$$Z^2 + Y^2 = X^2 \cot^2\gamma_1 \tag{5}$$

Subtracting and solving for X we find

$$X = \pm Z \frac{\sin\gamma_1}{\sin\gamma_2} \tag{6}$$

Substituting (6) into (4) we find

$$Y^2 = Z^2 \left[\frac{\cos^2\gamma_2}{\sin^2\gamma_2} - \frac{\sin^2\gamma_1}{\sin^2\gamma_2}\right]$$

$$\Rightarrow Y = \frac{\pm Z}{\sin\gamma_2} \sqrt{\cos^2\gamma_2 - \sin^2\gamma_1}$$

$$Z = Z \frac{\sin\gamma_2}{\sin\gamma_2}$$

$$\Rightarrow \hat{S} = \pm e_r \sin\gamma_1 + e_\Theta \sin\gamma_2 \pm e_\phi \sqrt{\cos^2\gamma_2 - \sin^2\gamma_1}$$

$$\hat{S} = \pm e_r \sin\gamma_1 + e_\Theta \sin\gamma_2 \pm e_\phi \sqrt{\cos^2\gamma_1 - \sin^2\gamma_2} \qquad (7)$$

1. if $\gamma_1>0,\ \gamma_2>0$

$$\hat{S} = e_r \sin\gamma_1 + e_\Theta \sin\gamma_2 + e_\phi \sqrt{\cos^2\gamma_1 - \sin^2\gamma_2}$$

$\hat{S}$ is in the octant e_r, e_Θ, e_ϕ

2. if $\gamma_1>0,\ \gamma_2<0$

$$\hat{S} = e_r \sin\gamma_1 + e_\Theta \sin\gamma_2 + e_\phi \sqrt{\cos^2\gamma_1 - \sin^2\gamma_2}$$

$\hat{S}$ is in the octant e_r, $-e_\Theta$, e_ϕ

3. if $\gamma_1<0,\ \gamma_2>0$

$$\hat{S} = e_r \sin\gamma_1 + e_\Theta \sin\gamma_2 + e_\phi \sqrt{\cos^2\gamma_1 - \sin^2\gamma_2}$$

$\hat{S}$ is in the octant $-e_r$, e_Θ, e_ϕ

4. if $\gamma_1<0,\ \gamma_1<0$

$$\hat{S} = e_r \sin\gamma_1 + e_\Theta \sin\gamma_2 + e_\phi \sqrt{\cos^2\gamma_1 - \sin^2\gamma_2}$$

$\hat{S}$ is in the octant $-e_r$, $-e_\Theta$, e_ϕ

For sensor D with output signature voltage 2.42^V and output angles δ_1 and δ_2 respectively, the sun vector can now be expressed as

$$\hat{S} = e_r \sin\delta_1 - e_\Theta \sin\delta_2 - e_\phi \sqrt{\cos^2\delta_1 - \sin^2\delta_2} \qquad (8)$$

This follows from the analysis for sensor C except that e_Θ is replaced by $-e_\Theta$ and e_ϕ by $-e_\phi$.

The reader can determine which quadrants the sun vector lies for (8) depending upon the conditions placed on δ_1 and δ_2.

Now, we will find $\hat{S}$ for sensors E and F. For sensor E with signature voltage $.86^V$, we have the following figure:

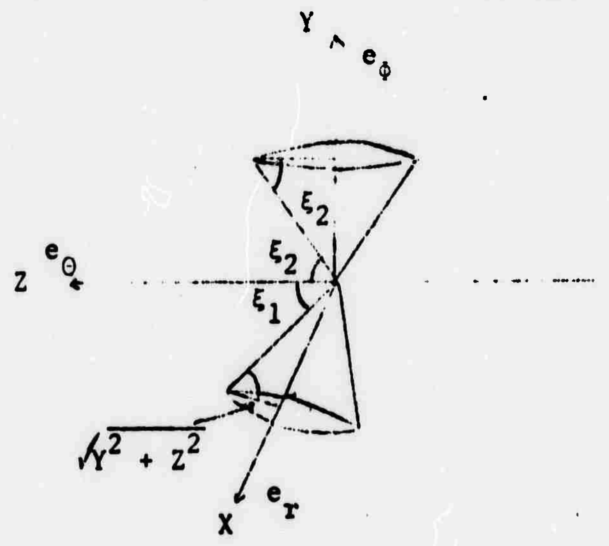


Figure 4

These cones actually intersect as can be seen from the figure for sensor C if we let $\bar{e}_\Phi = -e_\Theta$, $\bar{e}_\Theta = e_\Phi$, $\bar{e}_r = e_r$ where the bars represent the unit vectors for sensor E (and then remove the bars).

Sensor E faces positive e_Θ and ξ_1 and ξ_2 are the angles corresponding to the respective output voltages.

$$Y^2 + Z^2 = X^2 \cot^2 \xi_1 \tag{9}$$

$$X^2 + Z^2 = Y^2 \cot^2 \xi_2$$

Subtracting and solving for Y we get

$$Y = \pm X \frac{\sin\xi_2}{\sin\xi_1} \tag{10}$$

Substituting (10) into (9), we find

$$Z^2 = X^2 \frac{(\cos^2\xi_1 - \sin^2\xi_2)}{\sin^2\xi_1}$$

$$Z = \frac{\pm X}{\sin\xi_1} \sqrt{\cos^2\xi_1 - \sin^2\xi_2}$$

$$X = X \frac{\sin\xi_1}{\sin\xi_1}$$

The normalized sun vector can now be written as

$$\hat{S} = e_r \sin\xi_1 \pm e_\Theta \sqrt{\cos^2\xi_1 - \sin^2\xi_2} \pm e_\Phi \sin\xi_2$$

Due to output voltage #2 of sensor E as can be seen by the calibration of output voltage #2,

$$\hat{S} = e_r \sin\xi_1 + e_\Theta \sqrt{\cos^2\xi_1 - \sin^2\xi_2} + e_\Phi \sin\xi_2 \qquad (11)$$

Once again the reader can determine which quadrants the sun vector lies in for (11) depending upon the conditions placed on ξ_1 and ξ_2.

For sensor F with output signature voltage 1.57^V and output voltages f_1 and f_2 respectively.

$$\hat{S} = e_r \sin f_1 - e_\Theta \sqrt{\cos^2 f_1 - \sin^2 f_2} + e_\Phi \sin f_2 \qquad (12)$$

This follows from the analysis for sensor E except that

e_Θ is replaced by $-e_\Theta$ and e_Φ is not changed due to the remark preceding (11).

In summary, for each of the sun sensors, the unit vector $\hat{S}$ can be expressed as follows:

Sensor A $\quad \hat{S} = e_r \sqrt{\cos^2\alpha_1 - \sin^2\alpha_2} - e_\Theta \sin\alpha_2 + e_\Phi \sin\alpha_1$

Sensor B $\quad \hat{S} = -e_r \sqrt{\cos^2\beta_1 - \sin^2\beta_2} + e_\Theta \sin\beta_2 + e_\Phi \sin\beta_1$

Sensor C $\quad \hat{S} = e_r \sin\gamma_1 + e_\Theta \sin\gamma_2 + e_\Phi \sqrt{\cos^2\gamma_1 - \sin^2\gamma_2}$

Sensor D $\quad \hat{S} = e_r \sin\delta_1 - e_\Theta \sin\delta_2 - e_\Phi \sqrt{\cos^2\delta_1 - \sin^2\delta_2}$

Sensor E $\quad \hat{S} = e_r \sin\xi_1 + e_\Theta \sqrt{\cos^2\xi_1 - \sin^2\xi_2} + e_\Phi \sin\xi_2$

Sensor F $\quad \hat{S} = e_r \sin f_1 - e_\Theta \sqrt{\cos^2 f_1 - \sin^2 f_2} + e_\Phi \sin f_2$

To determine the angle between the sun vector and the axes of the satellite, we need only consider the respective dot products. That is:

$\hat{S}.e_r = \cos(\angle\hat{S}, e_r)$ = cosine of the angle between $\hat{S}$ and e_r.

$\hat{S}.e_\Theta = \cos(\angle\hat{S}, e_\Theta)$ = cosine of the angle between $\hat{S}$ and e_Θ.

$\hat{S}.e_\phi = \cos(\angle S, e_\phi)$ = cosine of the angle between $\hat{S}$ and e_ϕ.

Using this approach, we find the following angles:

	Sensor A	Sensor B	Sensor C
$(\angle\hat{S}, e_r)$	$\arccos\sqrt{\cos^2\alpha_1 - \sin^2\alpha_2}$	$\arccos(-\sqrt{\cos^2\alpha_1 - \sin^2\alpha_2})$	$90-\gamma_1$
$(\angle S, e_\Theta)$	$90 + \alpha_2$	$90-\beta_2$	$90-\gamma_2$
$(\angle S, e_\phi)$	$90 - \alpha_1$	$90-\beta_1$	$\arccos\sqrt{\cos^2\gamma_1 - \sin^2\gamma_2}$

	Sensor D	Sensor E	Sensor F
$(\angle S, e_r)$	$90-\delta_1$	$90+\delta_2$	$\arccos(-\sqrt{\cos^2\delta_1 - \sin^2\delta_2})$
$(\angle S, e_\Theta)$	$90-\xi_1$	$\arccos\sqrt{\cos^2\xi_1 - \sin^2\xi_2}$	$90-\xi_2$
$(\angle S, e_\phi)$	$90-f_1$	$\arccos(-\sqrt{\cos^2 f_1 - \sin^2 f_2})$	$90-f_2$

In each case output voltage #1 corresponds to the angle subscripted with a"1" and similarly for output voltage #2.

B. Results and Plots for Different Revolutions (Full Orbits and Real Time)

As can be seen by the following plots, the satellite is not well-behaved. During the later orbits, the motion of the satellite seems to becoming more stable than that of Rev. 480, however, at no time can a definite precession angle be found. The X axis of each of the plots represents seconds Greenwich Meridian time, and the Y axis represents the angle in degrees.

The program to determine the angles between the sun and the axes of the satellite is incorporated into the major program that will be discussed in Chapter V. The angles that are found range from 0° to 180°, so to predict a complete turn on either the pitch, roll, or yaw axis one would have to examine the appropriate plot. An example of this appears in the plot of $(\measuredangle S, e_\phi)$ for revolution 480. The sharp descent at approximately 53.4k seconds indicates a complete turn of the roll axis, i.e. $(\measuredangle S, e_\phi)$ is going from positive $0° \rightarrow 180°$ to negative $180° \rightarrow 0°$.

Since the signature voltages for the sun sensors were not extremely accurate, limits were set on each signature voltage to determine the appropriate sun sensor reading. These limits and the action taken can be found in the program referenced above.

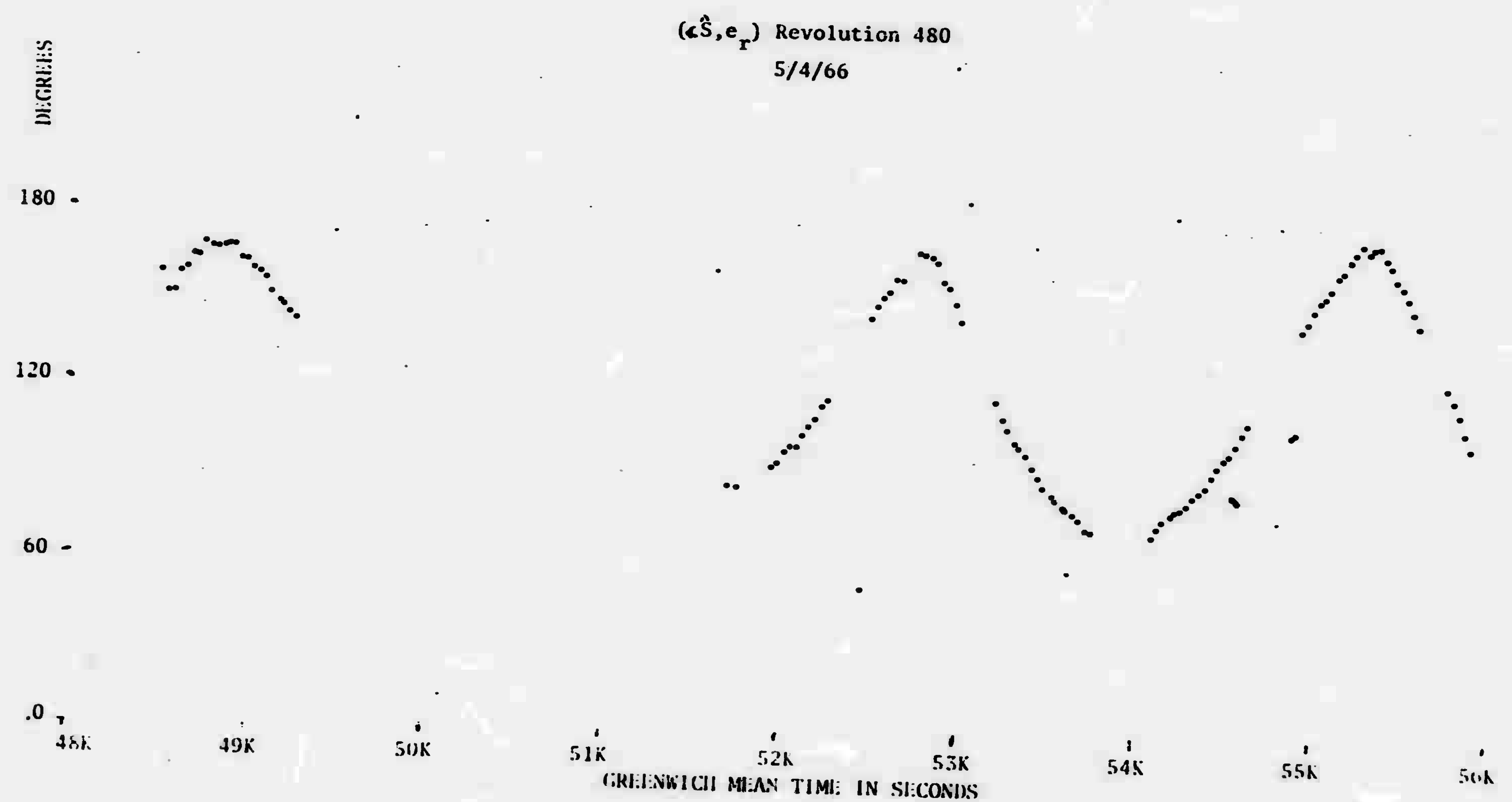

(≮Ŝ,e_r) Revolution 480
5/4/66
DEGREES
180
120
60
.0
48K
49K
50K
51K
52K
53K
54K
55K
56K
GREENWICH MEAN TIME IN SECONDS

Figure 5

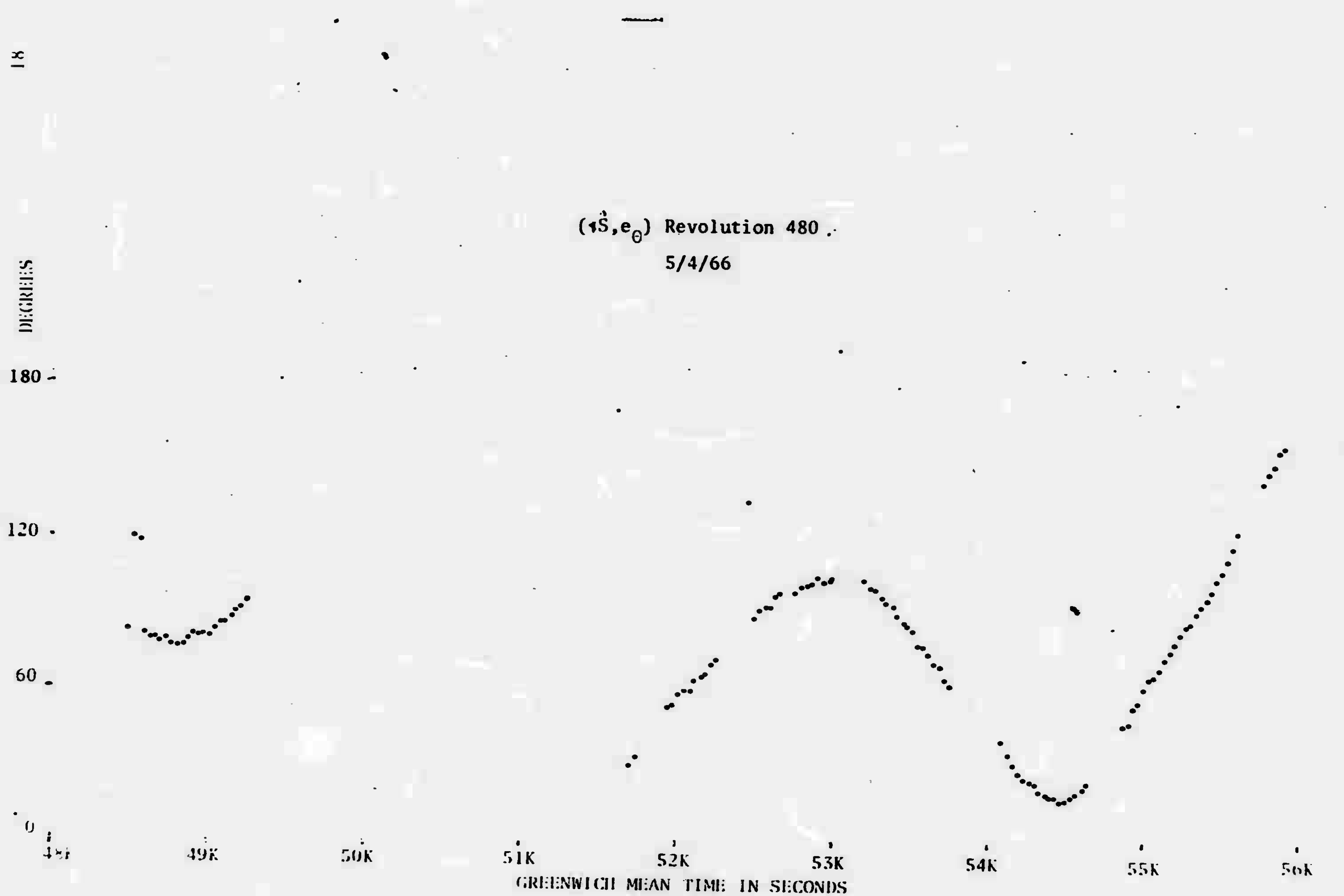

$(\dot{S}, e_\theta)$ Revolution 480
5/4/66
DEGREES
180
120
60
0
48K
49K
50K
51K
52K
53K
54K
55K
56K
GREENWICH MEAN TIME IN SECONDS

Figure 6

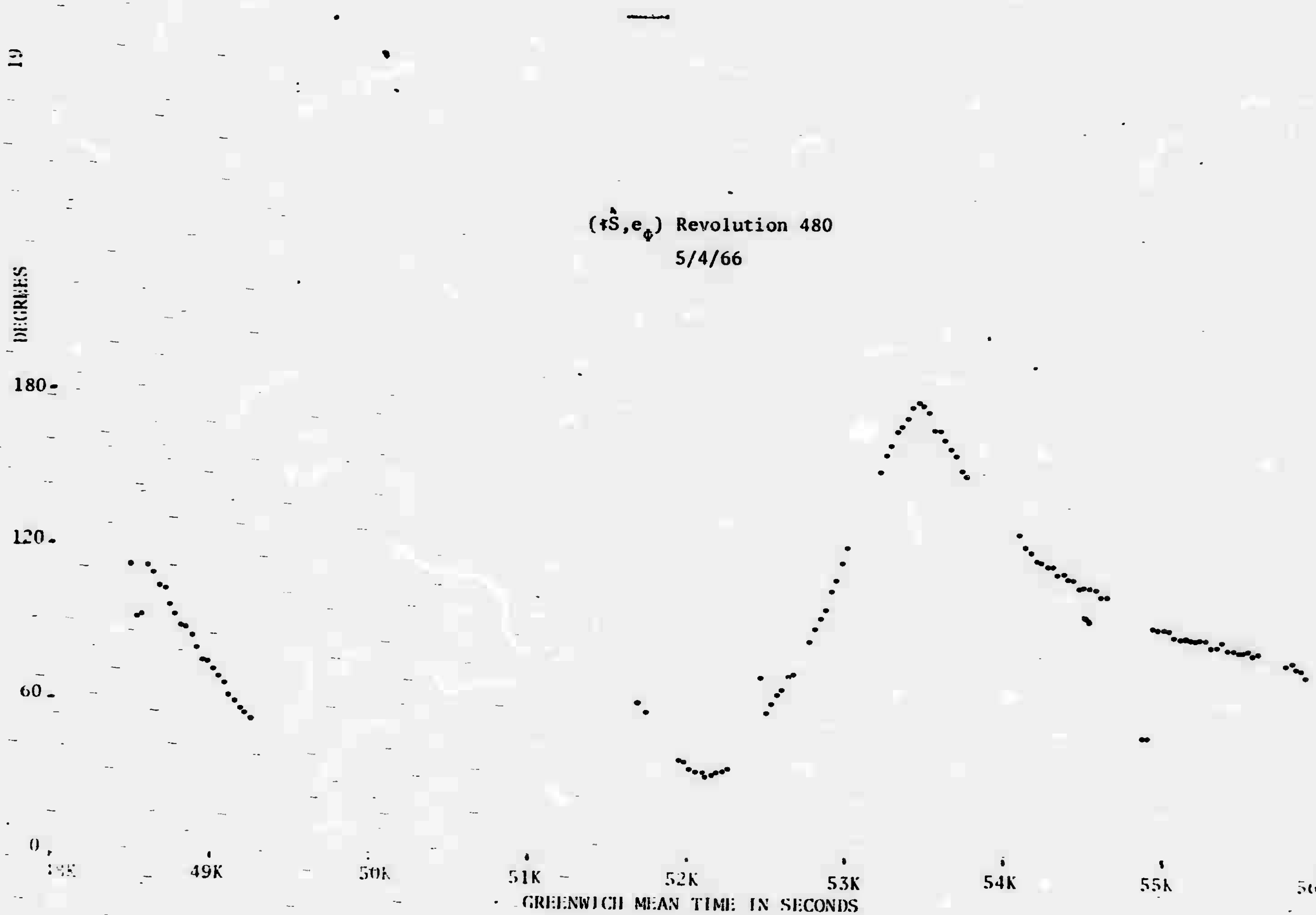
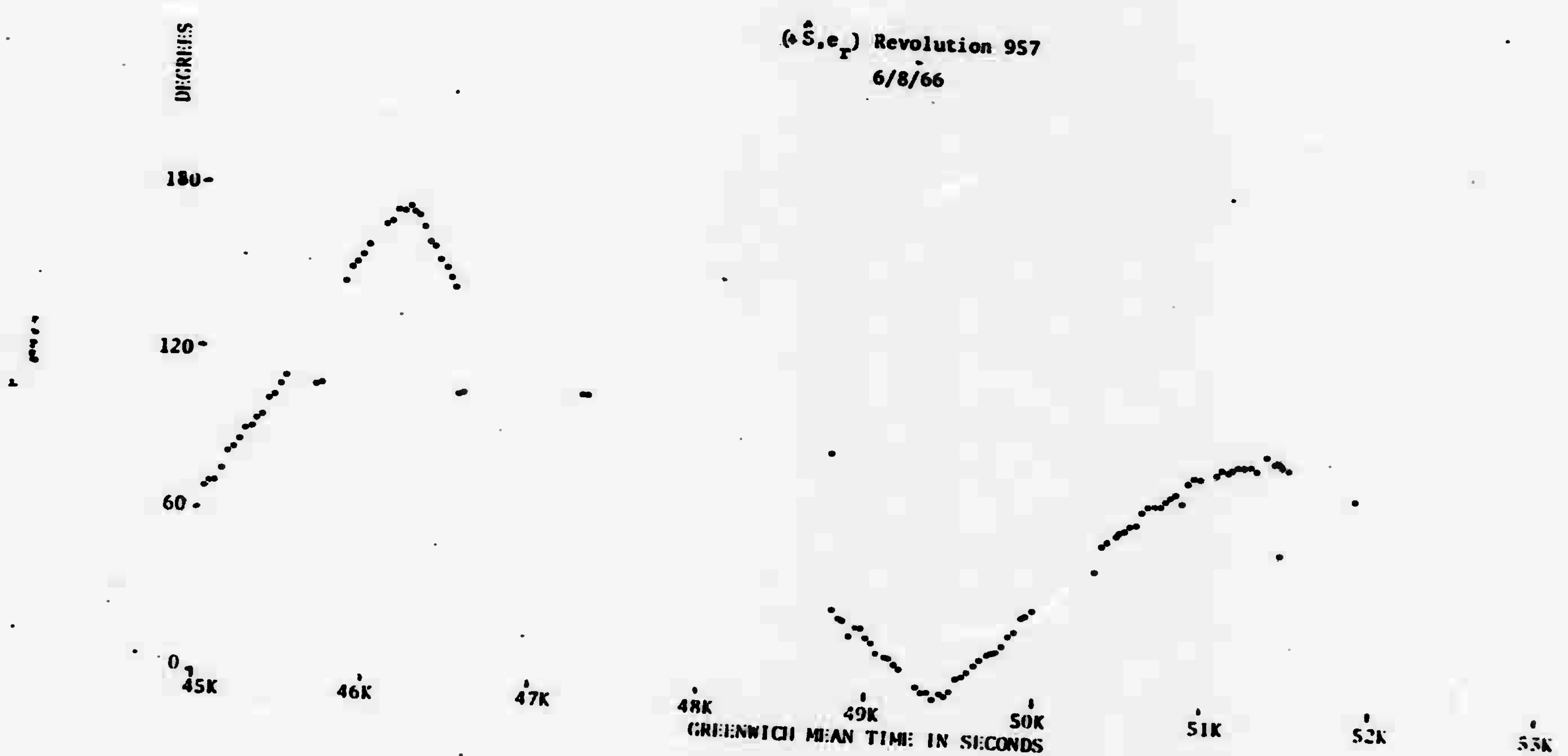

$(\dot{S}, e_\phi)$ Revolution 480
5/4/66
DEGREES
180
120
60
0
48K
49K
50K
51K
52K
53K
54K
55K
56K
GREENWICH MEAN TIME IN SECONDS

Figure 7



$(\angle\hat{S},e_r)$ Revolution 957
6/8/66
DEGREES
180
120
60
0
45K
46K
47K
48K
49K
50K
51K
52K
53K
GREENWICH MEAN TIME IN SECONDS

Figure 8

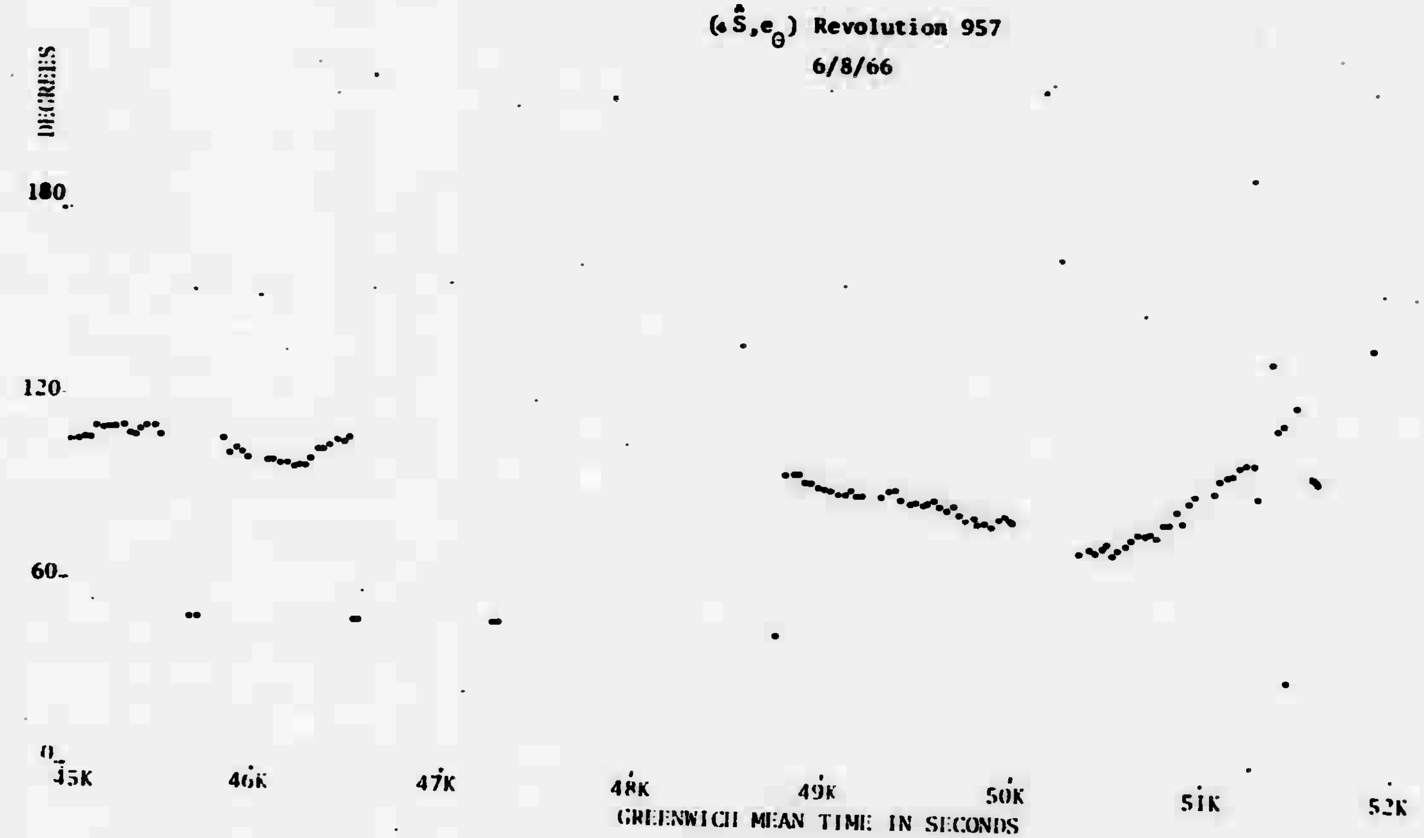

($\angle\hat{S},e_\theta$) Revolution 957
6/8/66
DEGREES
180
120
60
0
45K
46K
47K
48K
49K
50K
51K
52K
GREENWICH MEAN TIME IN SECONDS

Figure 9

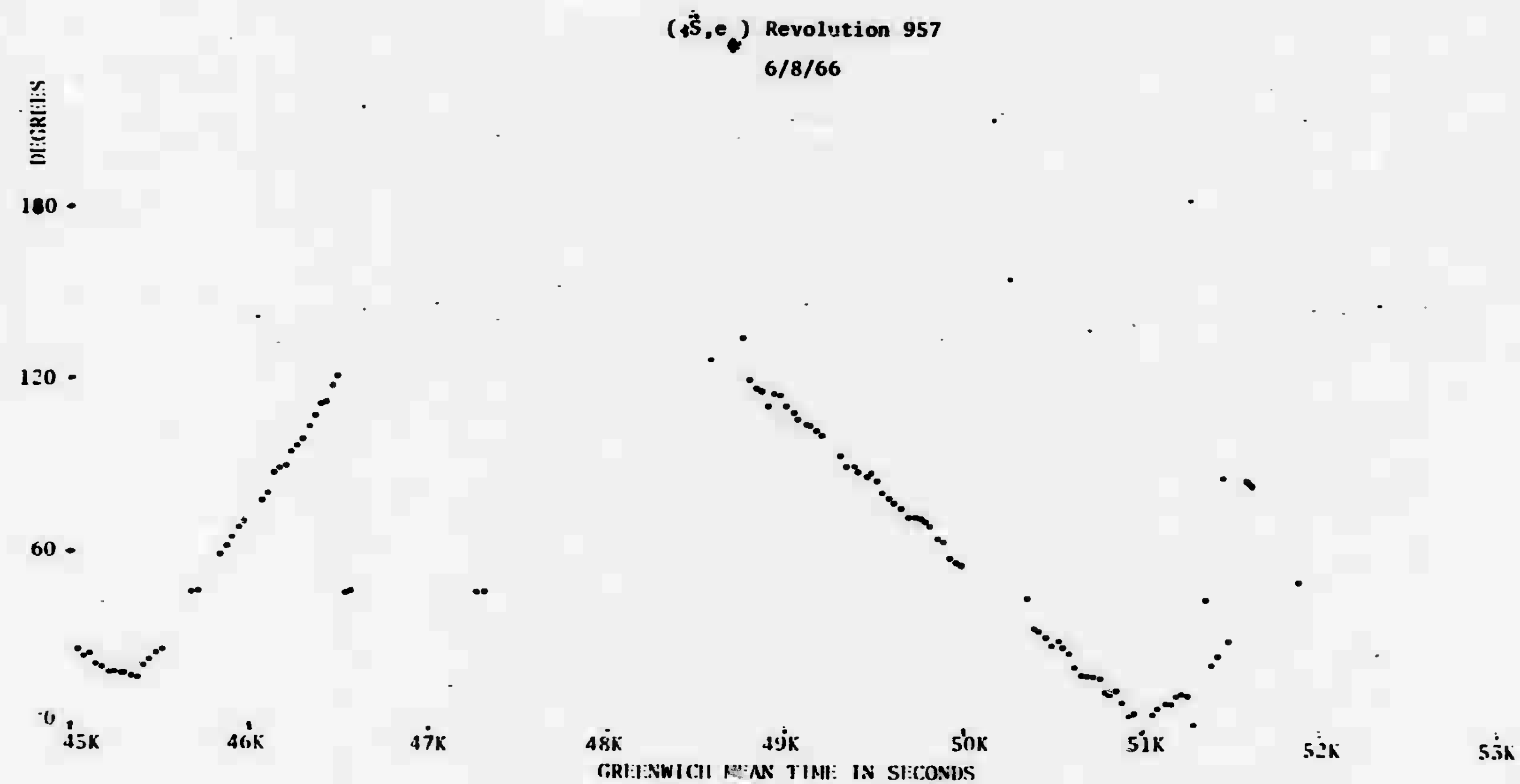
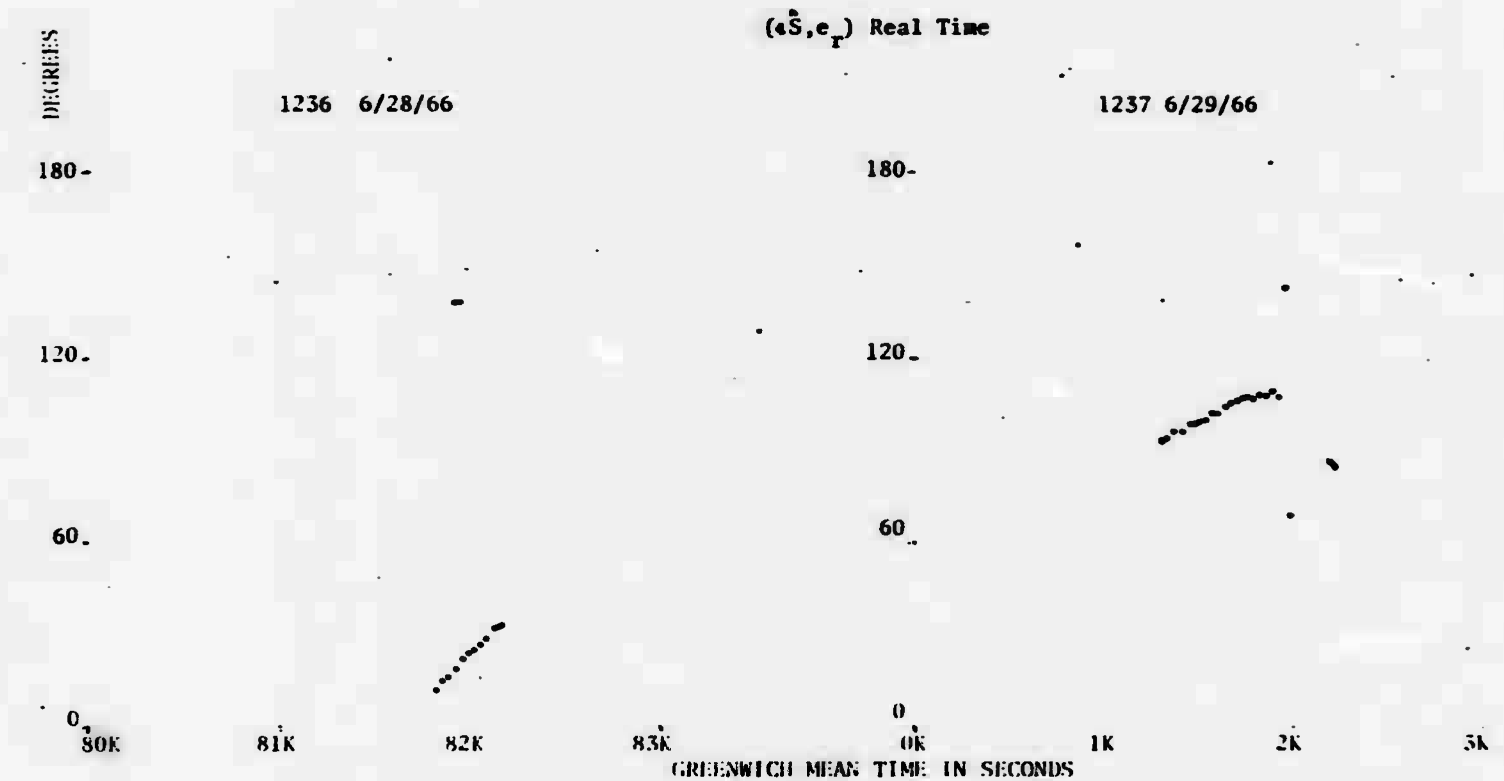

(ψS,e) Revolution 957
6/8/66
DEGREES
180
120
60
0
45K
46K
47K
48K
49K
50K
51K
52K
53K
GREENWICH MEAN TIME IN SECONDS

Figure 10



$(\angle \dot{S}, e_r)$ Real Time
DEGREES
1236 6/28/66
1237 6/29/66
180
120
60
0
80K
81K
82K
83K
0K
1K
2K
3K
GREENWICH MEAN TIME IN SECONDS

Figure 11

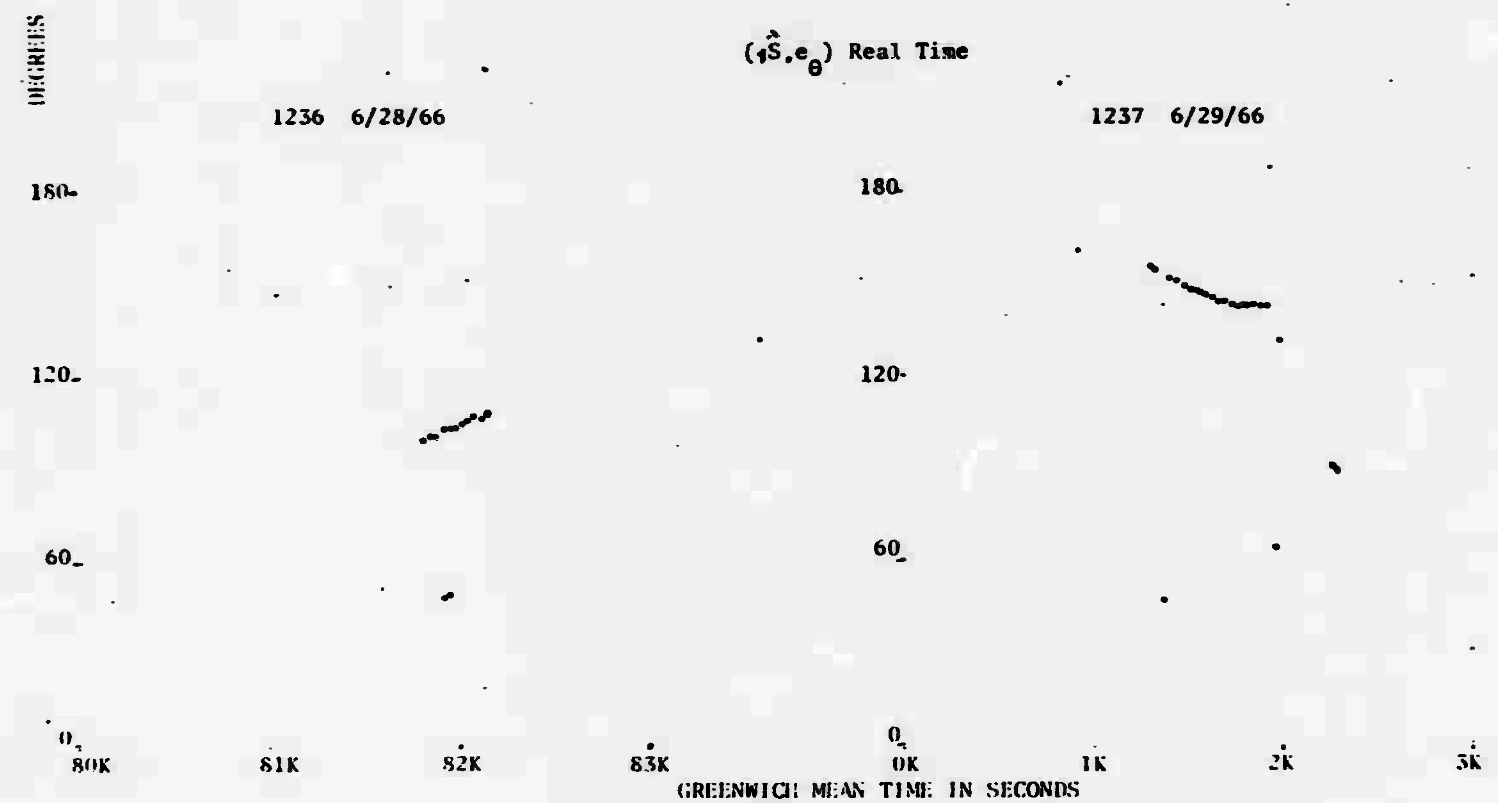
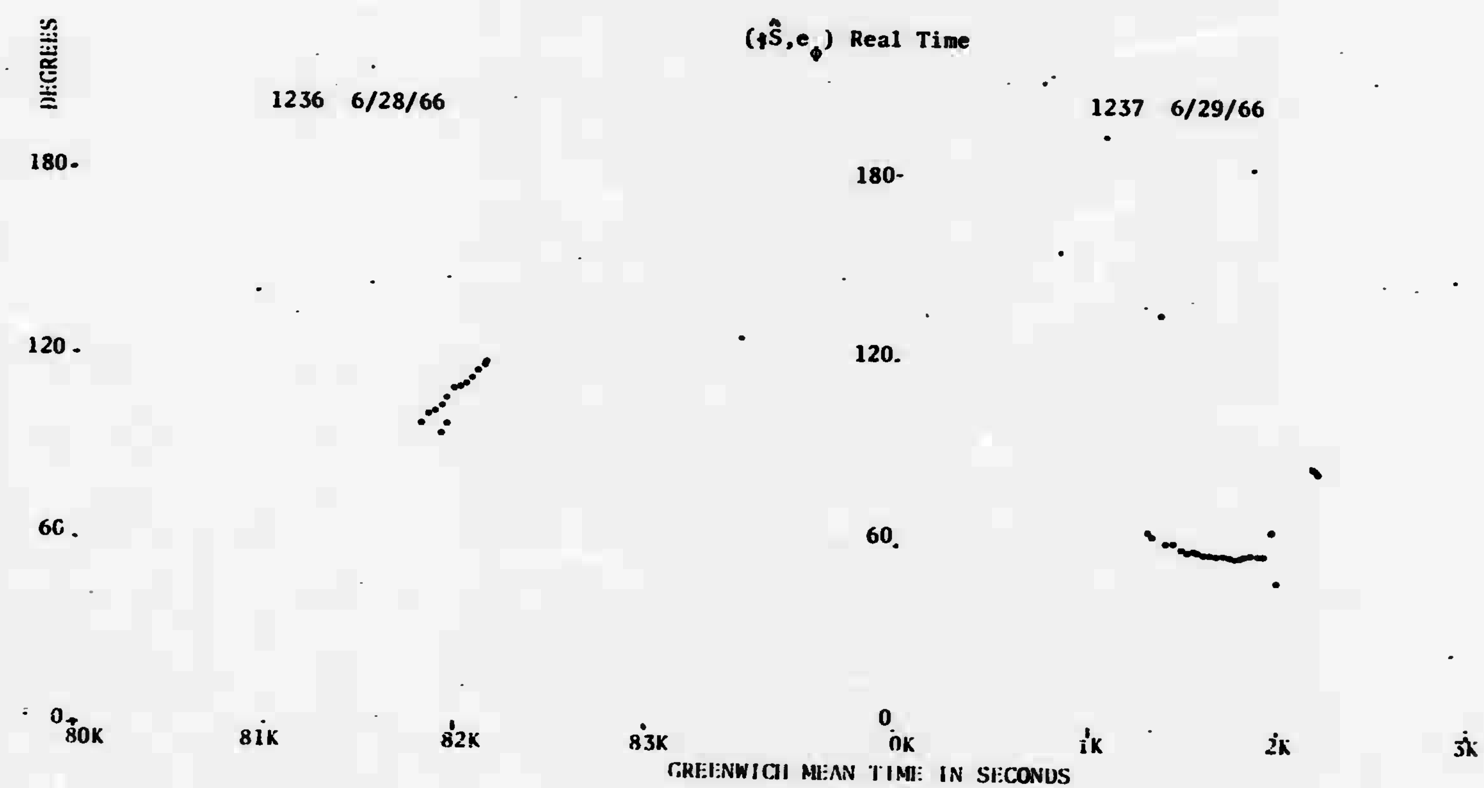
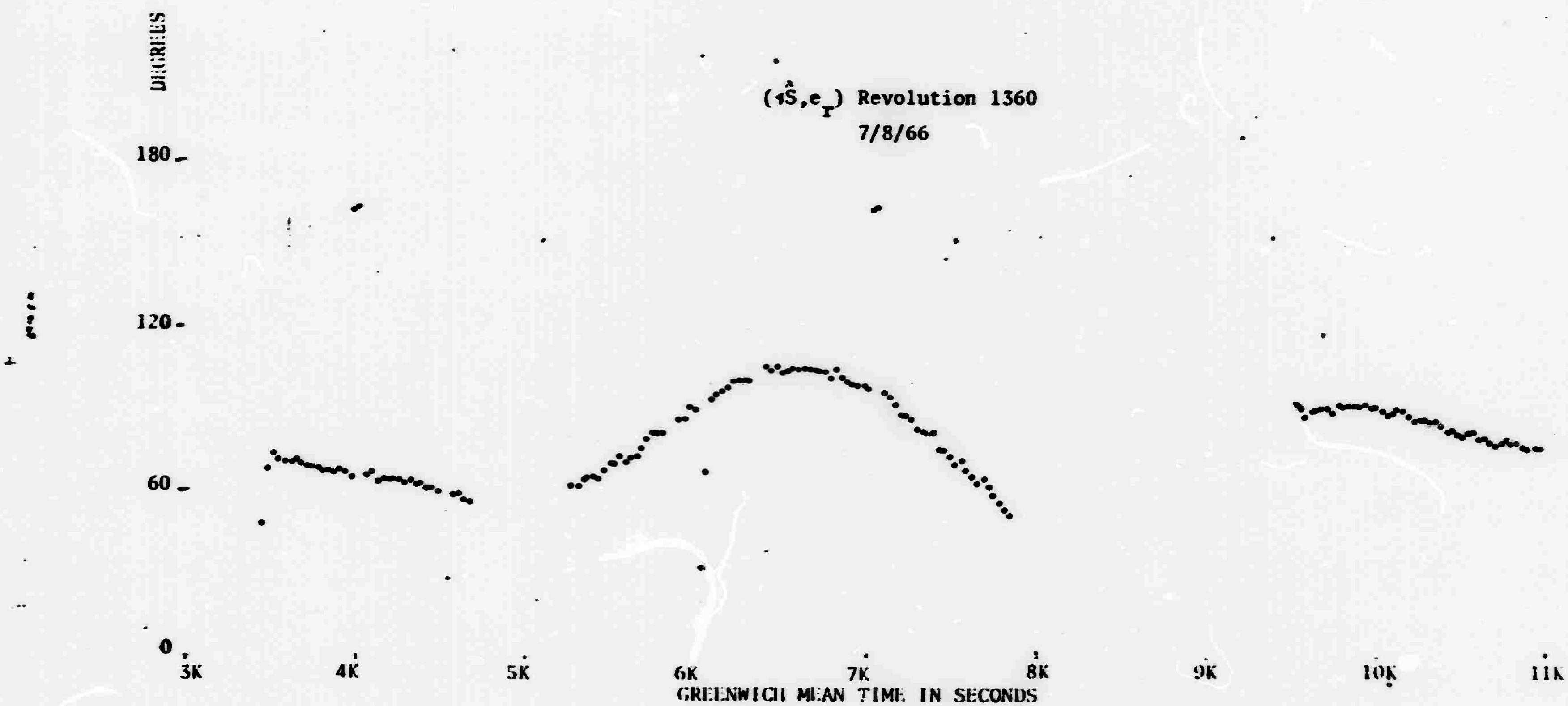

$({}_1\hat{S}, e_\theta)$ Real Time
DEGREES
1236 6/28/66
1237 6/29/66
180
120
60
0
80K
81K
82K
83K
0K
1K
2K
3K
GREENWICH MEAN TIME IN SECONDS

Figure 12



DEGREES
1236 6/28/66
($\hat{S}, e_\phi$) Real Time
1237 6/29/66
180
120
60
0
80K
81K
82K
83K
0K
1K
2K
3K
GREENWICH MEAN TIME IN SECONDS

Figure 13



$(\hat{S}, e_r)$ Revolution 1360
7/8/66
DEGREES
180
120
60
0
3K
4K
5K
6K
7K
8K
9K
10K
11K
GREENWICH MEAN TIME IN SECONDS

Figure 14

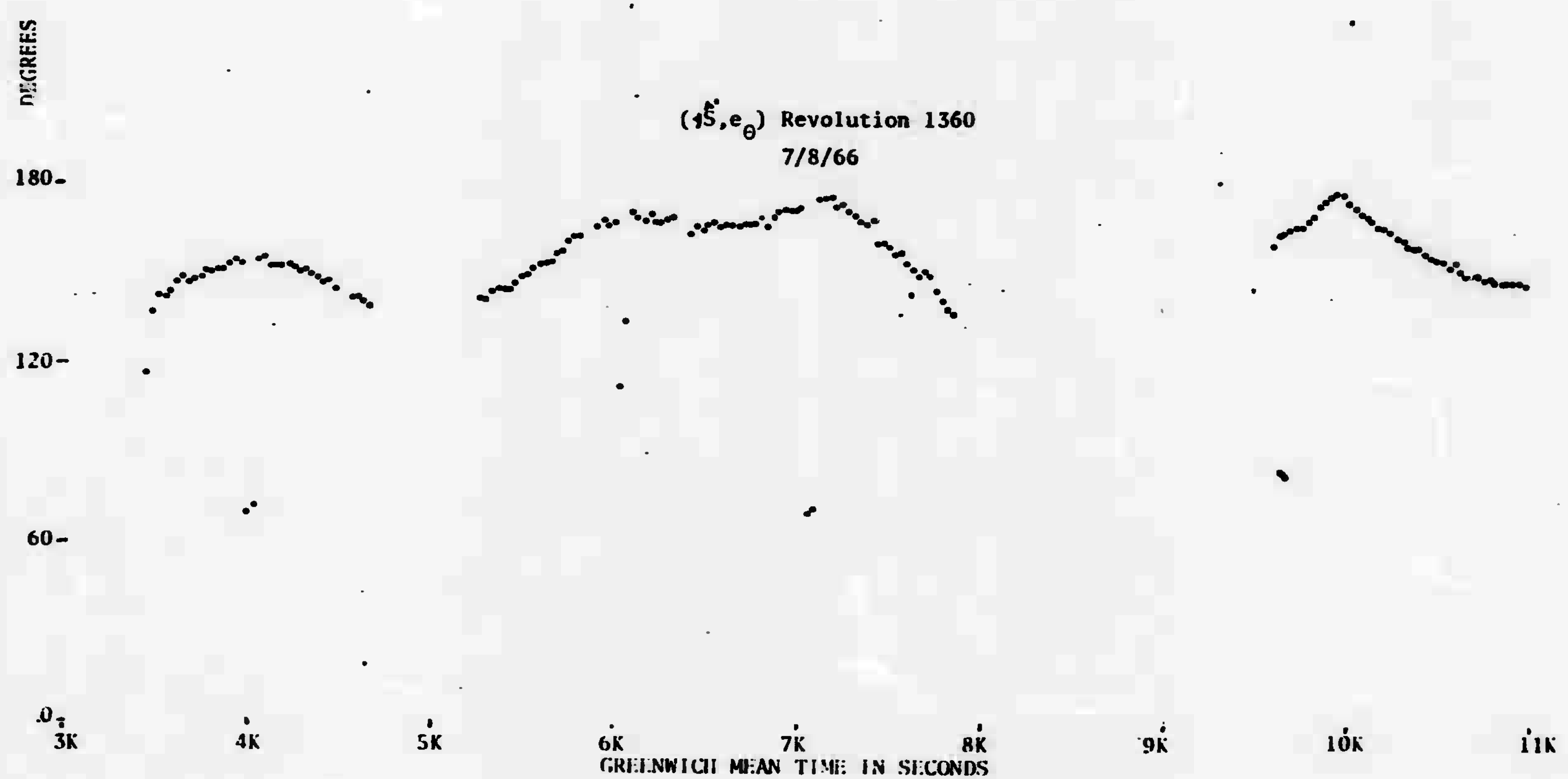

DEGREES
180
120
60
0
3K
4K
5K
6K
7K
8K
9K
10K
11K
GREENWICH MEAN TIME IN SECONDS
($\hat{S}$, e_θ) Revolution 1360
7/8/66

Figure 15

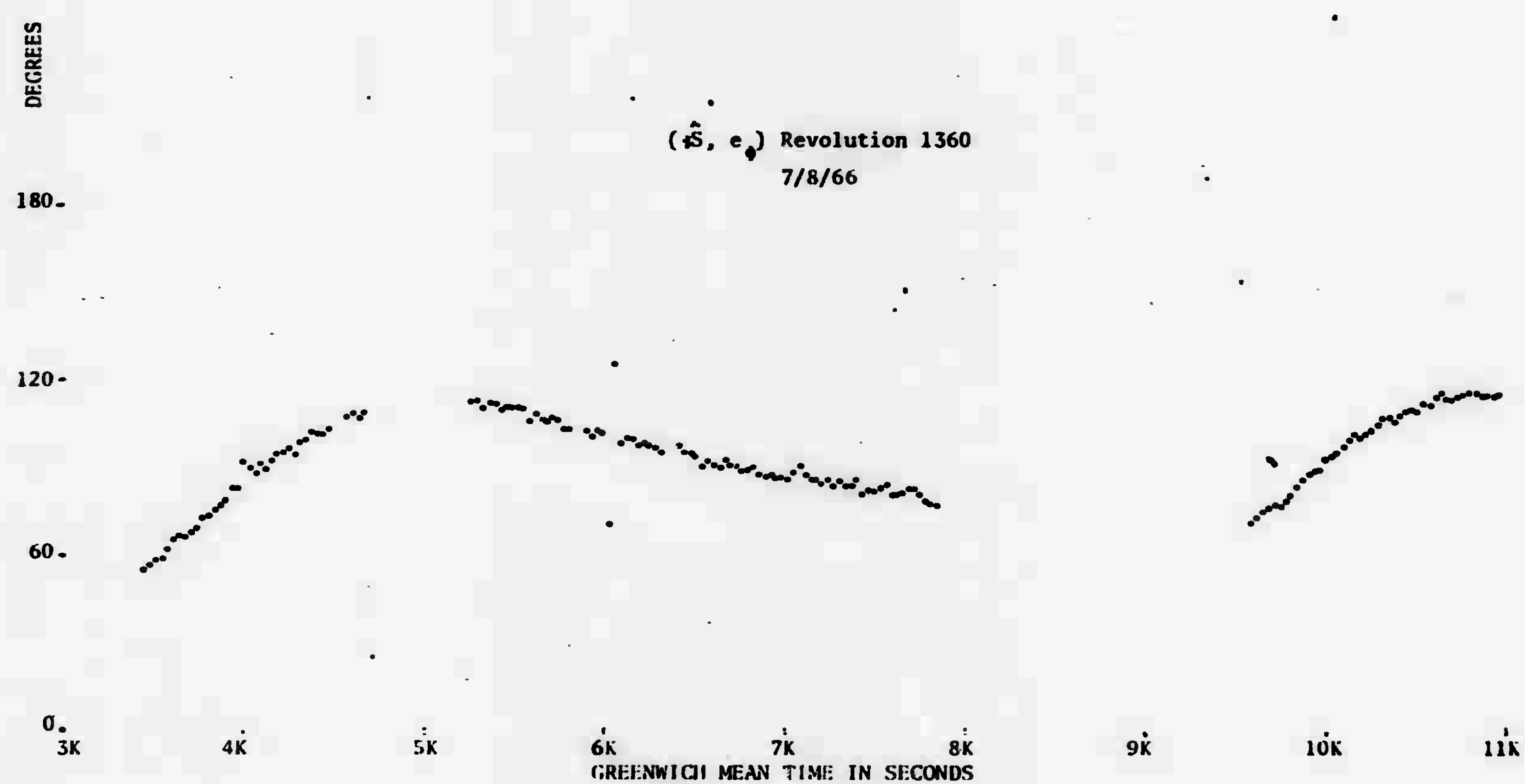

($\hat{S}$, e_ϕ) Revolution 1360
7/8/66
DEGREES
180
120
60
0
3K
4K
5K
6K
7K
8K
9K
10K
11K
GREENWICH MEAN TIME IN SECONDS

Figure 16

CHAPTER III

DESCRIPTION OF THE FIXED REFERENCE SYSTEM

A. Fixed Reference System with Respect to the Vernal Equinox

Since we are considering the motion of a satellite, the contribution of the earth's rotation about its axis and its rotation about the sun must be taken into account when trying to determine the aspect of the satellite with respect to a fixed system of coordinates. To express the aspect in terms of a geocentric system of coordinates would not have much meaning due to the above angular contributions. The fixed system used is with respect to the vernal equinox* (March 21).

Let i be a unit vector parallel to the line from the observer to the point on the celestial sphere where the sun appears at the time of the vernal equinox and directed toward the sun. Let k be a unit vector parallel to the polar axis of the earth and j = kxi

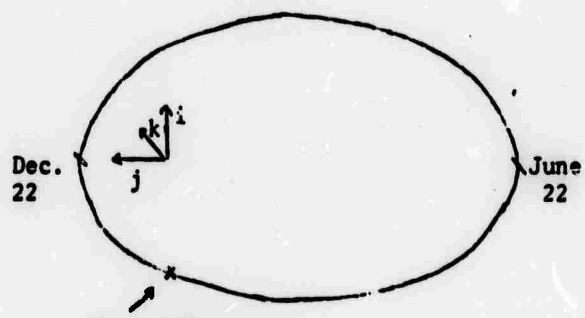


Figure 17

The unit vector j lies in the equatorial plane of the earth.

Every vector in this fixed system can be expressed in the form

$$\hat{V} = i \cos\Theta_V \cos\Phi_V + j \cos\Theta_V \sin\Phi_V + k \sin\Theta_V .$$

$\hat{V}$ is an arbitrary unit vector in this fixed system, Θ_V is the angle between the equatorial plane and the vector $\hat{V}$, ϕ_V is the azimuth of $\hat{V}$ with respect to the vernal equinox.

*The vernal equinox is defined as the intersection of the equatorial plane of the earth and orbital plane of the earth.

B. Expression of Required Vectors in this Fixed System of Coordinates

In this section we will describe the unit vectors $\hat{S}$, $\hat{M}$, e_r, $\hat{U}''$, and e_ϕ in the fixed system of base vectors i, j, k that will be used in the next three chapters.

1. The sun vector

The unit sun vector $\hat{S}$ from the earth to the sun can be expressed as

$$\hat{S} = i \cos\Theta_s \cos\Phi_s + j \cos\Theta_s \sin\Phi_s + k \sin\Theta_s \tag{13}$$

where Θ_s is the apparent declination of the sun and Φ_s is the apparent right ascension with respect to the vernal equinox at March 21. These angles were found in The American Ephemeris and Nautical Almanac - 1966.

2. The magnetic field vector

The unit vector $\hat{M}$ representing the magnetic field will be expressed as

$$\hat{M} = i \cos\Theta_H \cos\Phi_H + j \cos\Theta_H \sin\Phi_H + k \sin\Theta_H \tag{14}$$

The determination of these angles Θ_H and Φ_H for the fixed system of coordinates i, j, k will be discussed in chapter IV.

3. The axis of the satellite, e_r

The axis of the satellite can be expressed as the unit vector

$$e_r = i \cos\Theta \cos\Phi + j \cos\Theta \sin\Phi + k \sin\Theta \tag{15}$$

In Chapter V, we will go into a detailed description explaining how to obtain the angles Θ and Φ.

4. The vector $\hat{U}''$ from the center of the earth to the satellite

Given the following information:

tm = eppemeris transit time

Θ_e = latitude of the satellite from the ephemeris

Φ_e = longitude of the satellite from the ephemeris

t = Greenwich Meridian time in secs.

Φ_s = right ascension

Θ = angle between the nose axis of the satellite and the equatorial plane

Φ = azimuth of the nose axis W.R.t. the vernal equinox

$\omega = {}^{2\pi}/T$ where T is secs. in a sidereal day*

Given the angles Θ_E and Φ_E, we can set the unit vector $\hat{U}''$ in a rotating system as:

$$\hat{U}'' = \bar{i}\cos\Theta_E\cos\Phi_E + \bar{j}\cos\Theta_E\sin\Phi_E + \bar{k}\sin\Theta_E \tag{16}$$

where $\bar{i}$ is in the Greenwich Meridian Plane, $\bar{k}$ is in the direction of the north pole and $\bar{j} = \bar{k} \times \bar{i}$. Both $\bar{i}$ and $\bar{j}$ lie in the equatorial plane of the earth.

If we let tm equal the ephemeris transit time at which the Greenwich Meridian Plane transits the sun line (this time can be found on pages 19-33 in the above mentioned almanac), and $\omega = \frac{2\pi}{T}$ where T is the time in seconds for a sidereal day, and t = GMT in seconds, then

$$\bar{I} = i\cos[\omega(t-tm) + \Phi_s] + j\sin[\omega(t-tm) + \Phi_s] \tag{17}$$

$$\bar{J} = -i\sin[\omega(t-tm) + \Phi_s] + j\cos[\omega(t-tm) + \Phi_s] \tag{18}$$

Using (17) and (18) we can now express $\hat{U}''$ in the i, j, k fixed system, i.e.

$$\hat{U}'' = i\cos\Theta_E\cos[\omega(t-tm) + \Phi_s + \Phi_E] + j\cos\Theta_E\sin[\omega(t-tm) + \Phi_s + \Phi_E] + k\sin\Theta_E \tag{19}$$

where k is parallel to $\bar{k}$. Perhaps at this point, some diagrams might help clarify matters.

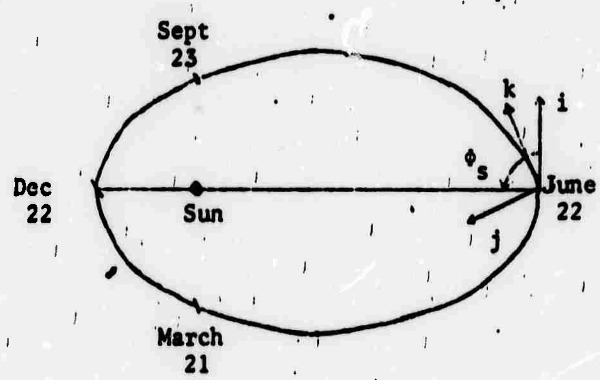


Figure 18

* A sidereal day is the duration of one rotation of the earth on its axis with respect to the vernal equinox. A sidereal day is 23 hours, 56 minutes, 4.09054 secs. of mean solar time.

In this diagram $j = k \times i$, and j lies in the equatorial plane of the earth along with i. Looking at the above figure from another view we have

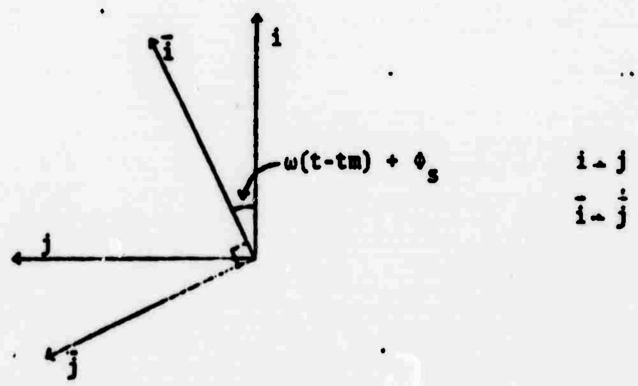


Figure 19

5. <u>The unit vector e_ϕ in the fixed system.</u>

Given the angles Θ and Φ any vector R along the nose axis of the satellite in the fixed system can be written as

$$R = r(i \cos\Theta \cos\Phi + j \cos\Theta \sin\Phi + k \sin\Theta)$$

$$\Rightarrow e_r = \frac{\partial R}{\partial r} = i \cos\Theta \cos\Phi + j \cos\Theta \sin\Phi + k \sin\Theta$$

$$\hat{N}_1 = \frac{1}{r}\frac{\partial R}{\partial \Theta} = -i \sin\Theta \cos\Phi - j \sin\Theta \sin\Phi + k \cos\Theta \qquad (20)$$

$$\hat{N}_2 = \frac{1}{r \cos\Theta}\frac{\partial R}{\partial \Phi} = -i \sin\Phi + j \cos\Phi \qquad (21)$$

and e_r, $\hat{N}_1$, N_2 defines an orthogonal system.

Since the satellite may be spinning, e_ϕ is in the plane of $\hat{N}_1$ and $\hat{N}_2$, and $e_\phi = \hat{N}_1 \cos\rho + \hat{N}_2 \sin\rho$.

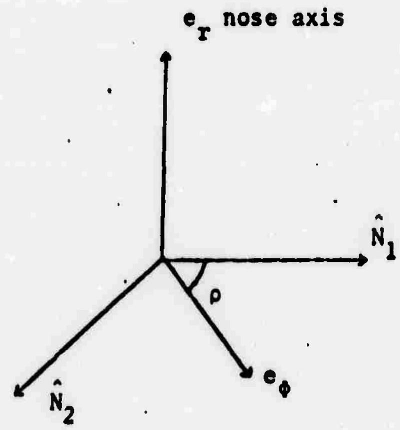


Figure 20

It should be noted that we could have let ρ' = angle between $\hat{N}_2$ and e_ϕ. Then we would get

$$e_\phi = \hat{N}_1 \sin\rho' + \hat{N}_2 \cos\rho' \quad .$$

Using the angle ρ we get

$$e_\phi = \hat{N}_1 \cos\rho + \hat{N}_2 \sin\rho = (-i \sin\Theta \cos\Phi - j \sin\Theta \sin\Phi + k \cos\Theta) \cos\rho$$

$$+ \sin\rho(-i \sin\Phi + j \cos\phi)$$

$$\Rightarrow e_\phi = -i(\sin\Theta \cos\Phi \cos\rho + \sin\Phi \sin\rho) - j(\sin\Theta \sin\Phi \cos\rho - \cos\Phi \sin\rho) \qquad (22)$$

$$+ k \cos\Theta \cos\rho \quad .$$

All that is needed to uniquely determine this unit vector is the angle ρ, and this will be discussed in Chapter VI.

CHAPTER IV

DETERMINATION OF Θ_H AND ϕ_H FOR A FIXED SYSTEM

In this chapter we will determine the magnetic field of the earth in a fixed system of coordinates. Using the same approach and same unit vectors as in (17) and (18) we can once again set up the following diagram:

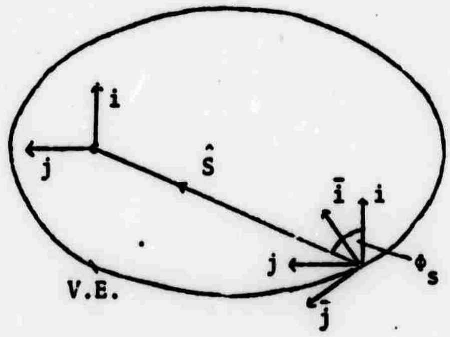


Figure 21

From the diagram we note that when $\bar{I} = \hat{S}$ the angle $(\measuredangle\bar{I},i) = \phi_s$ the right ascension. Thus the sun line and its right ascension now may be used as a reference in the representation of a vector initially expressed in the geocentric system of base vectors $\bar{I}$, $\bar{J}$, $\bar{K}$ in the fixed system i, j, k.

Let $\hat{M}$ be a unit vector initially expressed in the base $\bar{I}$, $\bar{J}$, $\bar{K}$.

$$\hat{M} = \bar{I}\cos\psi_H\cos\lambda_H + \bar{J}\cos\psi_H\sin\lambda_H + \bar{K}\sin\psi_H \tag{23}$$

where ψ_H is the angle between $\hat{M}$ and the equatorial plane of the earth, and λ_H is the azimuth of $\hat{M}$ with respect to the Greenwich Meridian Plane. Since $\bar{I}$ and $\bar{J}$ are in the equatorial plane of the earth we may write

$$\begin{aligned}\bar{I} &= i\cos(\omega t + \chi) + j\sin(\omega t + \chi)\\ \bar{J} &= -i\sin(\omega t + \chi) + j\cos(\omega t + \chi)\end{aligned} \tag{24}$$

If the time tm represents the time at which the Greenwich Meridian Plane transits the sun line, then

$$\bar{I} = i\cos\phi_s + j\sin\phi_s = i\cos(\omega tm + \chi) + j\sin(\omega tm + \chi)$$

$$\Rightarrow \chi = \phi_s - \omega tm \tag{25}$$

$$\text{and } \bar{i} = i \cos[\omega(t-tm) + \phi_s] + j \sin[\omega(t-tm) + \phi_s]$$

$$\bar{j} = -i \sin[\omega(t-tm) + \phi_s] + j \cos[\omega(t-tm) + \phi_s] \qquad (26)$$

which is the same as was shown in (17) and (18). Substituting (26) in (23), we get

$$\hat{M} = i \cos\psi_H \cos[\omega(t-tm) + \phi_s + \lambda_H] + j \cos\psi_H \sin[\omega(t-tm) + \phi_s + \lambda_H] + k \sin\psi_H \qquad (27)$$

However equating (27) and (14) we have

$$\sin\psi_H = \sin\Theta_H$$

$$\cos\psi_H \cos[\omega(t-tm) + \phi_s + \lambda_H] = \cos\Theta_H \cos\phi_H$$

$$\cos\psi_H \sin[\omega(t-tm) + \phi_s + \lambda_H] = \cos\Theta_H \sin\phi_H$$

$$\Rightarrow \psi_H = \Theta_H \text{ (angle between equatorial plane and } \hat{M}\text{)}$$

$$\Rightarrow \phi_H = \omega(t-tm) + \phi_s + \lambda_H \qquad (28)$$

Now using the program listed in the appendix,* the magnetic field can be expressed as

$$\hat{M} = e_{\Theta_E} \frac{X}{F} + e_{\phi_E} \frac{Y}{F} - e_{r_E} \frac{Z}{F} \qquad (29)$$

where r_E, Θ_E, ϕ_E are the Geocentric coordinates of the satellite.

In the rotating system the vector R_E is expressable as

$$R_E = [\bar{i} \cos\Theta_E \cos\phi_E + \bar{j} \cos\Theta_E \sin\phi_E + \bar{k} \sin\Theta_E] r_E$$

e_{ϕ_E} is positive east and is tangent to the circle of latitude,

*FOUGERE, P. private communication, L. G. Hanscom Field, Bedford, Mass.

e_{Θ_E} is tangent to the arc of the great circle going through the polar axes and in the direction of increasing latitude, e_{r_E} is from the center of the earth outward.

$$e_{r_E} = \frac{\partial R_E}{\partial r_E} \qquad e_{\Theta_E} = \frac{1}{r_E}\frac{\partial R_E}{\partial \Theta_c} \qquad e_{\Phi_c} = \frac{1}{r_E \cos\Theta_E}\frac{\partial R_E}{\partial \Phi_E}$$

In (29) the terms X, Y, Z and F are given by the referenced program and F equals the total field, X the component of the field in the e_{Θ_E} direction, Y the component of the field in positive e_{Φ_E}, and Z the component is radially downward.

Since the angles Θ_E and Φ_E are the latitude and longitude respectively of the satellite in the goecentric system (obtained from the ephemeris), we now have

$$\begin{aligned} e_{r_E} &= \bar{i} \cos\Theta_E \cos\Phi_E + \bar{j} \cos\Theta_E \sin\Phi_E + \bar{k} \sin\Theta_E \\ e_{\Theta_E} &= -\bar{i} \sin\Theta_E \cos\Phi_E - \bar{j} \sin\Theta_E \sin\Phi_E + \bar{k} \cos\Theta_E \\ e_{\Phi_E} &= -\bar{i} \sin\Phi_E + \bar{j} \cos\Phi_E \end{aligned} \tag{30}$$

Substituting (30) into (29) we find

$$\begin{aligned} F \hat{M} = &-\bar{i}[(X \sin\Theta_E + Z \cos\Theta_E) \cos\Phi_E + Y \sin\Phi_E] \\ &-\bar{j}[(X \sin\Theta_E + Z \cos\Theta_E) \sin\Phi_E - Y \cos\Phi_E] \\ &+\bar{k}[X \cos\Theta_E - Z \sin\Theta_E] \end{aligned} \tag{31}$$

where $F^2 = X^2 + Y^2 + Z^2$

From (23) and (28) we have

$$\hat{M} = \bar{i} \cos\Theta_H \cos\lambda_H + \bar{j} \cos\Theta_H \sin\lambda_H + \bar{k} \sin\Theta_H \tag{32}$$

Equating (31) and (32) we find

$$\begin{aligned} \Theta_H &= \arcsin\left[\frac{X \cos\Theta_E - Z \sin\Theta_E}{F}\right] \\ \lambda_H &= \arctan\left[\frac{Y \cos\Phi_E - (X \sin\Theta_E + Z \cos\Theta_E) \sin\Phi_E}{-Y \sin\Phi_E - (X \sin\Theta_E + Z \cos\Theta_E) \cos\Phi_E}\right] \end{aligned} \tag{33}$$

Substituting (33) into (27) with $\psi_H = \Theta_H$ i.e.

$$\hat{M} = i \cos\Theta_H \cos[\omega(t-tm) + \phi_s + \lambda_H] + j \cos\Theta_H \sin[\omega(t-tm) + \phi_s + \lambda_H] + k \sin\Theta_H \quad (34)$$

will give the unit vector in the direction of the earth magnetic field in the i, j, k fixed system. Equating (34) and (14) we now can find the angles Θ_H and Φ_H. The expression for angle Θ_H is given in (33) and

$$\Phi_H = \omega(t-tm) + \phi_s + \lambda_H \quad (35)$$

where λ_H is given also in (33). The magnetic field of the earth is then given by

$$M = F \hat{M}$$

where F is given by Fougere's program as a function of (r_E, Θ_E, Φ_E)

$$F^2 = X^2(r_E, \Theta_E, \Phi_E) + Y^2(r_E, \Theta_E, \Phi_E) + Z^2(r_E, \Theta_E, \Phi_E)$$

CHAPTER V

DETERMINATION OF Θ AND Φ FOR A FIXED SYSTEM

In this chapter we will discuss the determination of Θ, the angle between the nose axis of the satellite and the equatorial plane of the earth, and Φ, the aximuth of the nose axis of the satellite with respect to the vernal equinox.

A. Theoretical Description

The unit vector e_r in (15) can also be expressed in the form

$$e_r = \alpha\hat{M} + \beta\hat{S} + \frac{\lambda\hat{M} \times \hat{S}}{|\hat{M} \times \hat{S}|} \tag{36}$$

and dotting e_r with itself $\alpha^2 + \beta^2 + \gamma^2 + 2\alpha\beta\hat{M}\cdot\hat{S} = 1$ where $\hat{M}$ and $\hat{S}$ are as in (14) and (13) respectively. If we dot e_r with $\hat{M}$, $\hat{S}$, and $\hat{M} \times \hat{S}$ we find

$$\hat{M}\cdot e_r = \cos\beta_H = \alpha + \beta\hat{M}\cdot\hat{S} \tag{37}$$

$$\hat{S}\cdot e_r = \cos\beta_s = \alpha\hat{M}\cdot\hat{S} + \beta \tag{38}$$

$$\hat{M} \times \hat{S}\cdot e_r = \frac{\gamma(\hat{M}\times\hat{S})\cdot(\hat{M}\times\hat{S})}{|\hat{M}\times\hat{S}|} \tag{39}$$

Equation (39) can be rewritten as

$$\gamma = \frac{\hat{M} \times \hat{S}\cdot e_r}{|\hat{M}\times\hat{S}|} \tag{40}$$

The angle β_s is defined in Chapter II, and β_H is defined as

$$\cos\beta_H = \frac{H_z}{H_o} \tag{41}$$

where H_z is the component of the earth's magnetic field parallel to the nose axis of the satellite and H_o is the total magnetic field determined from the three magnetometers X, Y, and Z.

i.e.

$$H_o^2 = X^2 + Y^2 + Z^2 \tag{42}$$

Solving (37) and (38) for α and β by Cramer's Rule we find

$$\alpha = \frac{\cos\beta_H - \hat{M}\cdot\hat{S}\cos\beta_s}{1-(\hat{M}\cdot\hat{S})^2}$$

$$\beta = \frac{\cos\beta_s - \hat{M}\cdot\hat{S}\cos\beta_H}{1-(\hat{M}\cdot\hat{S})^2} \tag{43}$$

Let $1-(\hat{M}\cdot\hat{S})^2$ be replaced by $|\hat{M}x\hat{S}|^2$, then we can express e_r as

$$e_r = \frac{(\cos\beta_H - \hat{M}\cdot\hat{S}\cos\beta_s)\hat{M}}{|\hat{M}x\hat{S}|^2} + \frac{(\cos\beta_s - \hat{M}\cdot\hat{S}\cos\beta_H)\hat{S}}{|\hat{M}x\hat{S}|^2} + \frac{(\hat{M}\hat{S}e_r)\,\hat{M}x\hat{S}}{|\hat{M}x\hat{S}|^2} \tag{44}$$

Since $e_r\cdot k = \sin\Theta$ where e_r is as in (15), then using (44) for e_r and taking the scalar product of e_r and k,

$$\sin\Theta = \frac{(\cos\beta_H-\hat{M}\cdot\hat{S}\cos\beta_s)\sin\Theta_H+(\cos\beta_s-\hat{M}\cdot\hat{S}\cos\beta_H)\sin\Theta_s+(\hat{M}\hat{S}e_r)\cos\Theta_H\cos\Theta_s\sin(\phi_s-\phi_H)}{1-(\hat{M}\cdot\hat{S})^2} \tag{45}$$

All terms in (45) are now known except for the triple scalar product for $(\hat{M}\hat{S}e_r)$. If we take the scalar product of (44) with e_r we get

$$1 = \frac{(\cos\beta_H - \hat{M}\cdot\hat{S}\cos\beta_s)\cos\beta_H + (\cos\beta_s - \hat{M}\cdot\hat{S}\cos\beta_H)\cos\beta_s + (\hat{M}\hat{S}e_r)^2}{|\hat{M}x\hat{S}|^2} \tag{46}$$

Replacing $|\hat{M}x\hat{S}|^2$ by $1-(\hat{M}\cdot\hat{S})^2$, (46) can be rewritten as

$$(\hat{M}\hat{S}e_r)^2 = 1-(\hat{M}\cdot\hat{S})^2 - \cos^2\beta_H - \cos^2\beta_s + 2\hat{M}\cdot\hat{S}\cos\beta_s\cos\beta_H \tag{47}$$

Replacing $\cos^2\beta_H$ by $1-\sin^2\beta_H$ and completing the square, (47) can be rewritten as

$$(\hat{M}\hat{S}e_r)^2 = \sin^2\beta_H - \cos^2\beta_s - (\hat{M}\cdot\hat{S})^2 + 2\hat{M}\cdot\hat{S}\cos\beta_s\cos\beta_H$$
$$- \cos^2\beta_s\cos^2\beta_H + \cos^2\beta_s\cos^2\beta_H$$

$$(\hat{M}\hat{S}e_r)^2 = \sin^2\beta_H - \cos^2\beta_s(1-\cos^2\beta_H) - (\hat{M}\cdot\hat{S} - \cos\beta_s\cos\beta_H)^2$$

$$(\hat{M}\hat{S}e_r)^2 = \sin^2\beta_H\sin^2\beta_s - (\hat{M}\cdot\hat{S} - \cos\beta_s\cos\beta_H)^2 \tag{48}$$

Now replacing (48) into (45) the angle Θ can be determined, however an ambiguity arises due to the term

$$(\hat{M}\hat{S}e_r) = \pm\sqrt{\sin^2\beta_H\sin^2\beta_s - (\hat{M}\cdot\hat{S} - \cos\beta_s\cos\beta_H)^2} \tag{49}$$

According to Report AFCRL-65-516 page 5, the positive sign for $(\hat{M}\hat{S}e_r)$ must be taken where e_r is on the same side of the $\hat{M}$-$\hat{S}$ plane as $\hat{M}x\hat{S}$, and the negative sign where e_r is on the opposite side of the $\hat{M}$-$\hat{S}$ plane. Also for a flight where the magnetometer data is accurate $(\hat{M}\hat{S}e_r)$ will be real, i.e.

$$(\hat{M}\hat{S}e_r)^2 = \sin^2\beta_H\sin^2\beta_s - (\hat{M}\,\hat{S} - \cos\beta_s\cos\beta_H)^2 > 0$$

In actual flight, however, it occurred that $(\hat{M}\hat{S}e_r)^2 < 0$ which is physically impossible. This was a result of erroneous magnetic field data which occurred frequently during the flight of OV1-5.

When e_r makes an angle <90° with the vector $\hat{M}x\hat{S}$, then $(\hat{M}\hat{S}e_r) > 0$ and $(\hat{M}\hat{S}e_r) < 0$ when the angle made >90°. In the case of a rocket flight, this presents no problem but for a satellite we are unable to predetermine the position of e_r relative to the $\hat{M}$-$\hat{S}$ plane. In order to get around this problem, the output for angles Θ and Φ from the program ASPECT was analyzed and those values which gave the smoothest curve fit were selected.

In this manner we determined whether to take $+(\ddot{M}Se_r)$ or $-(\ddot{M}Se_r)$ during a specific position of the flight or a combination of both. That is, when a crossover in the plot of Θ occurs the sign of $(\ddot{M}Se_r)$ should be examined to insure a smooth fit. If at any time in the flight $\beta_s = 0$, the ambiguity does not exist since at this time

$$\Theta = \Theta_s$$

$$\text{and} \qquad \Phi = \Phi_s$$

as can be seen in revolution 957.

Using expressions (14) and (13) and forming the scalar product of each with e_r we have

$$\cos\Theta \cos\Theta_H \cos(\Phi-\Phi_H) + \sin\Theta_H \sin\Theta = \cos\beta_H \tag{50}$$

$$\cos\Theta \cos\Theta_s \cos(\Phi-\Phi_s) + \sin\Theta_s \sin\Theta = \cos\beta_s \tag{51}$$

Now according to Report AFCRL-63-871 page 5 and 6, upon multiplying (50) by $\sin\Theta_s$ and (51) by $\sin\Theta_H$ we can eliminate $\sin\Theta$ from (50) and (51). In a similar manner $\cos\Theta$ can be eliminated and the above equations can be rewritten as

$$\begin{aligned} &b_1 \cos\Theta \cos\Phi + b_2 \cos\Theta \sin\Phi = a \\ &(b_1 \sin\Theta - c) \cos\Phi + (b_2 \sin\Theta - c_2) \sin\Phi = 0 \end{aligned} \tag{52}$$

where

$$a = \cos\beta_H \sin\Theta_s - \cos\beta_s \sin\Theta_H$$

$$b_1 = \cos\Theta_H \sin\Theta_s \cos\Phi_H - \cos\Theta_s \sin\Theta_H \cos\Phi_s$$

$$b_2 = \cos\Theta_H \sin\Theta_s \sin\Phi_H - \cos\Theta_s \sin\Theta_H \sin\Phi_s$$

$$c_1 = \cos\Theta_H \cos\beta_s \cos\Phi_H - \cos\Theta_s \cos\beta_H \cos\Phi_s$$

$$c_2 = \cos\Theta_H \cos\beta_s \sin\Phi_H - \cos\Theta_s \cos\beta_H \sin\Phi_s$$

the solution of (52) is

$$\sin\Phi = \frac{a(b_1 \sin\Theta - c_1)}{\cos\Theta(b_1 c_2 - b_2 c_1)}$$

$$\cos\Phi = \frac{-a}{\cos\Theta}\left(\frac{b_2 \sin\Theta - c_2}{b_1c_2-b_2c_1}\right)$$

or $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ (53)

$$\tan\Phi = \frac{b_1\sin\Theta - c_1}{-(b_2\sin\Theta - c_2)}$$

Another and less tedious method of determining the angle Φ is to take the scalar product of e_r in (15) and (44), both with i and j. In this case

$$\tan\Phi = \frac{j\cdot e_r}{i\cdot e_r} \qquad (54)$$

where the expression for e_r in (54) is that found in (44).

B. Program to Determine Θ and Φ with Explanations

The following Fortran program named ASPECT was written for the IBM 7094 computer and its purpose is to calculate the angle Θ and Φ as described in (45) and (53) respectively. Frequently during the flight of OV1-5, we found poor magnetic field data. As a result, when running this program consideration of the angle $\beta_H = (\hat{M}, e_r)$ sometimes produced erroneous values. That is, terms in which $\cos\beta_H$, written in this program as CBETAH, were involved produced at times negative values for

```
1. - STHETP * STHETP
1. - STHETN * STHETN
```

used for the determination of

```
CTHETP = SQRT(1.-STHETP * STHETP)
```

and

```
CTHETN = SQRT(1.-STHETN * STHETN) .
```

In ASPECT, CTHETP = COSΘ when the plus sign for $(\hat{M}\hat{S}e_r)$ was used and CTHETN = COSΘ when the minus sign for $(\hat{M}\hat{S}e_r)$ was used. In the final analysis these few poor data points were overlooked to insure a good curve fit.

The output of this program is as follows:

$$\hat{M}\hat{S}e_rt = \sin^2\beta_H \sin^2\beta_s - (\hat{M}\cdot\hat{S} - \cos\beta_s \cos\beta_H)^2$$

$$\hat{M}\hat{S}e_r = \sqrt{\hat{M}\hat{S}e_rt}$$

THETAP and PHIP are the angles Θ and Φ when $+\sqrt{\hat{M}\hat{S}e_r t}$ was used

THETAN and PHIN are the angles Θ and Φ when $-\sqrt{\hat{M}\hat{S}e_r t}$ was used

H = the total magnetic field determined from the data

SUNAX = $(\hat{S}, e_r)$

SUNTHE= $(\hat{S}, e_\Theta)$

SUNPHI= $(\hat{S}, e_\Phi)$

GMTT = Greenwich Mean time of each data point

THETNW and PHINEW are the latitude and longitude respectively of the satellite with respect to Greenwich for the time GMTT.

SIG is the signature voltage of that sun sensor which is giving sun data

```
$ID       0232    MARCOTTE     ASPECT              E3                    44
$ICP              TIME=02,PAGES=15
$ALL              CONTINUE
$IBJOB            DLOGIC
$IBFTC ASPECT     LIST,REF,DECK,SDD
C        ASPECT
C        THETA=ANGLE BETWEEN AXIS AN+ EQUATORIAL PLANE
C        PHI=AZIMUTH OF AXIS OF SATELLITE WRT VERNAL EQUINOX
C        THETA-H=ANGLE BETWEEN EQUATORIAL PLANE AND MAGNETIC FIELD
C        PHI-H=AZIMUTH OF MAGNETIC FIELD WRT VERNAL EQUINOX
         DIMENSION THETA(258),PHI(258),X(258),Y(258),Z(258),F(258),GMT(258)
         OMEGA=2.*3.1415927/86164.091
      95 READ (5,90) THETAS,PHIS,TM,NUMB,CHECKA,SPAN
      90 FORMAT (2F10.5,F10.2,6X,I4,2F5.1)
         J=0
         J=J+1
         THETAS=THETAS*.0174533
         PHIS=PHIS*.0174533
         READ (5,2) (THETA(I),PHI(I),X(I),Y(I),Z(I),F(I),GMT(I),I=1,NUMB)
       2 FORMAT (2F10.3,10X,5F10.3)
       4 READ (5,1) GMTT,XX,YY,ZZ,SA1,SA2,SIG,MTEST
       1 FORMAT (7F10.3,5X,I5)
C        DETERMINATION OF SUNAX,SUNPHI,SUNTHE
         A=ABS(SIG-4.69)
         B=ABS(SIG-3.90)
         C=ABS(SIG-3.19)
         D=ABS(SIG-2.47)
         E=ABS(SIG-.86)
         FF=ABS(SIG-1.57)
         IF(.31-A) 5,10,10
       5 IF (.35-B) 6,11,11
       6 IF (.35-C) 7,12,12
       7 IF(.40-D) 8,13,13
       8 IF (.35-FF) 9,14,14
       9 IF(.35-E) 97,15,15
      10 ALPHA1=-45.113-1.273*SA1+14.399*SA1*SA1-2.167*SA1**3
         ALPHA2=-44.852+1.363*SA2+13.063*SA2*SA2-1.984*SA2**3
         SUNPHI=90.-ALPHA1
         SUNTHE=90.+ALPHA2
         ALPHA1=ALPHA1*.0174533
         ALPHA2=ALPHA2*.0174533
         XCOS=SQRT(COS(ALPHA1)**2-SIN(ALPHA2)**2)
         XSIN=SQRT(1.-XCOS*XCOS)
         SUNAX=ATAN(XSIN/XCOS)
         SUNAX=SUNAX*57.29578
         GO TO 16
      11 BETA1=-45.956+4.636*SA1+11.970*SA1*SA1-1.901*SA1**3
         BETA2=-45.291+3.724*SA2+12.014*SA2*SA2-1.873*SA2**3
         SUNPHI=90.-BETA1
         SUNTHE=90.-BETA2
         BETA1=BETA1*.0174533
         BETA2=BETA2*.0174533
         XCOS=-SQRT(COS(BETA1)**2-SIN(BETA2)**2)
         XSIN=SQRT(1.-XCOS*XCOS)
         SUNAX=ATAN(XSIN/XCOS)
         SUNAX=SUNAX*57.29578+180.
         GO TO 16
      12 GAMMA=-45.877+2.570*SA1+12.816*SA1*SA1-1.994*SA1**3
         SUNAX=90.-GAMMA
         GAMMA2=-46.958+4.022*SA2+12.134*SA2*SA2-1.907*SA2**3
         SUNTHE=90.-GAMMA2
```

```
      GAMMA=GAMMA*.0174533
      GAMMA2=GAMMA2*.0174533
      XCOS=SQRT(COS(GAMMA)**2-SIN(GAMMA2)**2)
      XSIN=SQRT(1.-XCOS*XCOS)
      SUNPHI=ATAN(XSIN/XCOS)
      SUNPHI=SUNPHI*57.29578
      GO TO 16
   13 DELTA=-45.795+3.296*SA1+12.528*SA1*SA1-1.970*SA1**3
      SUNAX=90.-DELTA
      DELTA2=-45.594+3.668*SA2+12.100*SA2*SA2-1.885*SA2**3
      SUNTHE=90.-DELTA2
      DELTA=DELTA*.0174533
      DELTA2=DELTA2*.0174533
      XCOS=-SQRT(COS(DELTA)**2-SIN(DELTA2)**2)
      XSIN=SQRT(1.-XCOS*XCOS)
      SUNPHI=ATAN(XSIN/XCOS)*57.29578+180.
      GO TO 16
   14 FTHETA=-45.601+2.523*SA1+12.483*SA1*SA1-1.919*SA1**3
      SUNAX=90.-FTHETA
      FTHET2=-46.021+3.728*SA2+12.227*SA2**2-1.895*SA2**3
      SUNPHI=90.-FTHET2
      FTHETA=FTHETA*.0174533
      FTHET2=FTHET2*.0174533
      XCOS=-SQRT(COS(FTHETA)**2-SIN(FTHET2)**2)
      XSIN=SQRT(1.-XCOS*XCOS)
      SUNTHE=ATAN(XSIN/XCOS)*57.29578+180.
      GO TO 16
   15 EPSILN=-48.057+14.876*SA1+5.154*SA1*SA1-.851*SA1**3
      SUNAX=90.-EPSILN
      EPSIL2=46.387-7.424*SA2-9.191*SA2**2+1.400*SA2**3
      SUNPHI=90.-EPSIL2
      EPSILN=EPSILN*.0174533
      EPSIL2=EPSIL2*.0174533
      XCOS=SQRT(COS(EPSILN)**2-SIN(EPSIL2)**2)
      XSIN=SQRT(1.-XCOS*XCOS)
      SUNTHE=ATAN(XSIN/XCOS)*57.29578
C     DETERMINATION OF BETA-H
   16 XMG=249.458*XX-604.989
      YMG=251.256*YY-606.030
      ZMG=247.934*ZZ-592.562
      H=SQRT(XMG*XMG+YMG*YMG+ZMG*ZMG)
      GO TO 30
   26 J=J+1
   30 REFER=GMTT-GMT(J)
      IF (REFER) 23,24,24
   23 REFER=-REFER
   24 IF (REFER-CHECKA) 25,25,26
   25 TIME=GMTT-GMT(J)
      TIMET=TIME/SPAN
      IF (TIME) 27,28,29
   27 FNEW=F(J)-TIMET*(F(J-1)-F(J))
      XNEW=X(J)-TIMET*(X(J-1)-X(J))
      YNEW=Y(J)-TIMET*(Y(J-1)-Y(J))
      ZNEW=Z(J)-TIMET*(Z(J-1)-Z(J))
      THETNW=THETA(J)-TIMET*(THETA(J-1)-THETA(J))
      PHINEW=PHI(J)-TIMET*(PHI(J-1)-PHI(J))
      GO TO 33
   28 FNEW=F(J)
      XNEW=X(J)
      YNEW=Y(J)
      ZNEW=Z(J)
```

```
      THETNW=THETA(J)
      PHINEW=PHI(J)
      GO TO 33
   22 FNEW=F(J)-TIMEI*(F(J)-F(J+1))
      XNEW=X(J)-TIMEI*(X(J)-X(J+1))
      YNEW=Y(J)-TIMEI*(Y(J)-Y(J+1))
      ZNEW=Z(J)-TIMEI*(Z(J)-Z(J+1))
      THETNW=THETA(J)-TIMEI*(THETA(J)-THETA(J+1))
      PHINEW=PHI(J)-TIMEI*(PHI(J)-PHI(J+1))
   33 CBETAH=XMG/H
      THETNW=THETNW*.0174533
      PHINEW=PHINEW*.0174533
C     DETERMINATION OF THETA-H AND PHI-H
      STHETH=(XNEW*COS(THETNW)-ZNEW*SIN(THETNW))/FNEW
      CTHETH=SQRT(1.-STHETH*STHETH)
      THETAH=ATAN(STHETH/CTHETH)
      TERM=XNEW*SIN(THETNW)+ZNEW*COS(THETNW)
      AMDAB=ATAN((YNEW*COS(PHINEW)-TERM*SIN(PHINEW))/(-YNEW*
     1SIN(PHINEW)-TERM*COS(PHINEW)))
      PHIH=OMEGA*(GMTI-TM)+PHIS-AMDAB
C     DETERMINATION OF MS AND MSER
      ABC=COS(THETAS)
      DEF=SIN(THETAS)
      ACF=COS(PHIS)
      DFH=SIN(PHIS)
      EMDOTS=CTHETH*ABC*COS(PHIH-PHIS)+STHETH*DEF
      GUNAX=SUNAX*.0174533
      SINSUN=SIN(GUNAX)
      COSUN=COS(GUNAX)
      COSBET=COSUN*CBETAH
      EMSERT=SINSUN*SINSUN*(1.-CBETAH*CBETAH)-(EMDOTS-COSBET)**2
      IF (EMSERT) 61,61,62
   61 EMSER=0.0
      GO TO 63
   62 EMSER=SQRT(EMSERT)
C     DETERMINATION OF THETA AND PHI
   63 AA=(DEF-EMDOTS*STHETH)*COSUN+(STHETH-EMDOTS*DEF)*CBETAH
      BB=ABC*CTHETH*SIN (PHIS-PHIH)*EMSER
      CC=1.-EMDOTS*EMDOTS
      STHETP=(AA+BB)/CC
      STHETN=(AA-BB)/CC
      CTHETP=SQRT(1.-STHETP*STHETP)
      THETAP=ATAN(STHETP/CTHETP)*57.29578
      CTHETN=SQRT(1.-STHETN*STHETN)
      THETAN=ATAN(STHETN/CTHETN)*57.29578
      CPHIH=COS(PHIH)
      SPHIH=SIN(PHIH)
      B1=CTHETH*DEF*CPHIH-ABC*STHETH*ACF
      B2=CTHETH*DEF*SPHIH-ABC*STHETH*DFH
      C1=CTHETH*COSUN*CPHIH-ABC*CBETAH*ACF
      C2=CTHETH*COSUN*SPHIH-ABC*CBETAH*DFH
      ANUM=B1*STHETP-C1
      ADENOM=-(B2*STHETP-C2)
      BNUM=B1*STHETN-C1
      BDENOM=-(B2*STHETN-C2)
      PHIP=ATAN (ANUM/ADENOM)*57.29578
      PHIN=ATAN (BNUM/BDENOM)*57.29578
      IF(PHIP) 72,72,72
   72 IF(ANUM) 73,75,75
   73 PHIP=PHIP+180.
      GO TO 76
```

```
   7. IF(ANUM) 71,73,73
   71 PHIP=PHIP+360.
      GO TO 76
75    PHIP=PHIP
76    IF(PHIN) 80,82,82
82    IF(BNUM) 83,85,85
   83 PHIN=PHIN+180.
      GO TO 44
   80 IF (BNUM) 81,83,83
   81 PHIN=PHIN+360.
      GO TO 44
   85 PHIN=PHIN
   44 THETNW=THETNW*57.29578
      PHINEW=PHINEW*57.29578
      WRITE(6,49) EMSER,EMSERT,H,THETAP,THETAN,PHIP,PHIN,SUNAX,SUNTHE,
     2SUNPHI,THETNW,PHINEW,SIG,GMIT
   49 FORMAT(1X,2F7.4,10F6.1,F6.3,F10.3)
      PUNCH 43,H,THETAP,THETAN,PHIP,PHIN,SUNAX,SUNTHE,SUNPHI,THETNW,
     1PHINEW,GMTT
   43 FORMAT(10F6.1,F10.3)
   97 IF (MTEST-999) 4,95,96
   96 CALL EXIT
      STOP
      END
$DATA
```

CHAPTER VI

DETERMINATION OF $(\sphericalangle e_\phi, \hat{U}'')$

In this chapter we will determine the angle between the unit vector $\hat{U}''$ as expressed in (19) and e_ϕ in (22). The unit vector e_ϕ is the direction of sensor C on the satellite and $e_\phi = -e_r \times e_\theta$ for the e_r, e_θ, e_ϕ system with regards to the satellite as discussed in Chapter II.

A. Determination of e_ϕ in another fixed system

The problem as was mentioned in Chapter III is to solve for the angle ρ. In order to do this, we must set up another system for e_ϕ. In the following figure, e_ϕ is in the plane of $\hat{N}_1$ and $\hat{N}_2$.

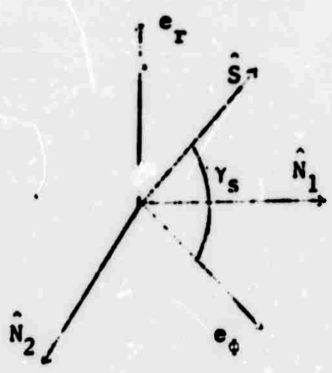


Figure 22

and can be expressed in the form

$$e_\phi = \alpha\hat{S} + \frac{\beta(e_r \times \hat{S})}{|e_r \times \hat{S}|} + \frac{\gamma\hat{S}\times(e_r \times \hat{S})}{|\hat{S}\times(e_r \times \hat{S})|} \tag{55}$$

where $\hat{S}$, and the vectors $\dfrac{(e_r \times \hat{S})}{|e_r \times \hat{S}|}$, $\dfrac{\hat{S} \times (e_r \times \hat{S})}{|\hat{S} \times (e_r \times \hat{S})|}$

define an orthogonal system of unit vectors. Since

$$\hat{S} \times (e_r \times \hat{S}) = e_r\,(\hat{S}\cdot\hat{S}) - \hat{S}(e_r\cdot\hat{S}) = e_r - \hat{S}\cos\beta_s$$

it is easy to show that

$$|e_r \times \hat{S}| = |\hat{S} \times (e_r \times \hat{S})| = \sin\beta_s \tag{56}$$

where $\beta_s = (\sphericalangle S\ e_r)$

To determine α, β, and γ in (55) we must take the scalar product of e_ϕ with $\hat{S}$, $\dfrac{e_r \times \hat{S}}{\sin\beta_s}$, and $\dfrac{\hat{S} \times (e_r \times \hat{S})}{\sin\beta_s}$ respectively. This gives us:

$$\alpha = e_\phi . \hat{S} = \cos\gamma_s \tag{57}$$

$$\beta = e_\phi . \frac{(e_r \times \hat{S})}{\sin\beta_s} = \frac{(e_\phi e_r \hat{S})}{\sin\beta_s} \tag{58}$$

$$\gamma = e_\phi . \frac{(\hat{S} \times (e_r \times \hat{S}))}{\sin\beta_s} = e_\phi . \frac{[e_r - \hat{S}\cos\beta_s]}{\sin\beta_s} = -\frac{\cos\gamma_s \cos\beta_s}{\sin\beta_s} \tag{59}$$

Using (57), (58), (59) it is now possible to set

$$e_\phi = \cos\gamma_s\ \hat{S} + \frac{(e_\phi e_r \hat{S})}{\sin\beta_s}\ \frac{(e_r \times \hat{S})}{\sin\beta_s} - \frac{\cos\gamma_s \cos\beta_s}{\sin\beta_s}\ \frac{\hat{S} \times (e_r \times \hat{S})}{\sin\beta_s} \tag{60}$$

If we consider now the scalar product of e_ϕ in (60) with itself,

$$1 = \cos^2\gamma_s + \frac{(e_\phi e_r \hat{S})^2}{\sin^2\beta_s} + \frac{\cos^2\gamma_s \cos^2\beta_s}{\sin^2\beta_s}$$

$$\Rightarrow \sin^2\beta_s = \sin^2\beta_s \cos^2\gamma_s + \cos^2\beta_s \cos^2\gamma_s + (e_\phi e_r \hat{S})^2$$

$$\Rightarrow (e_\phi e_r \hat{S}) = \pm\sqrt{\sin^2\beta_s - \cos^2\gamma_s} \tag{61}$$

Inserting (61) into (60), e_ϕ is now expressable in the orthogonal system $\hat{S}$, $\dfrac{e_r \times \hat{S}}{\sin\beta_s}$, $\dfrac{\hat{S} \times (e_r \times \hat{S})}{\sin\beta_s}$. The plus sign for $(e_\phi e_r \hat{S})$ will used when e_ϕ is on the same side of the $e_r - \hat{S}$ plane as $e_r \times \hat{S}$. Otherwise the minus sign will be used in (61).

B. Determination of the angle ρ

Using expression (22), (60) with the value of $(e_\theta e_r \hat{S})$ found in (61), the purpose of this section will be to determine the angle ρ that e_ϕ makes with $\hat{N}_1$ while rotating in the $\hat{N}_1 - \hat{N}_2$ plane. In this method, we shall equate the k coefficients in each of the above mentioned expressions for e_ϕ. The k coefficient of (22) is $\cos\Theta \cos\rho$. If we let

$$\begin{aligned} A &= \cos\Theta \sin\Phi \sin\Theta_s - \cos\Theta_s \sin\Phi_s \sin\Theta \\ B &= \cos\Theta_s \cos\Phi_s \sin\Theta - \cos\Theta \cos\Phi \sin\Theta_s \\ C &= \cos\Theta \cos\Theta_s \sin(\Phi_s - \Phi) \end{aligned} \tag{62}$$

It can be shown that the k coefficient of (60) where $\hat{S}$ and e_r are in the i, j, k systems is

$$\cos\gamma_s \sin\Theta_s + \frac{C(e_\phi e_r \hat{S})}{\sin^2\beta_s} - \frac{\cos\gamma_s \cos\beta_s}{\sin^2\beta_s} \left[\cos\Theta_s \cos\Phi_s B - \cos\Theta_s \sin\Phi_s A\right] \tag{63}$$

Equating $\cos\Theta \cos\rho$ to (63) and simplifying

$$\cos\rho = \frac{\sin^2\beta_s \cos\gamma_s \sin\Theta_s + C(e_\phi e_r \hat{S}) - \cos\gamma_s \cos\beta_s \cos\Theta_s \left[\cos\Theta_s \sin\Theta - \cos\Theta \sin\Theta_s \cos(\Phi_s - \Phi)\right]}{\sin^2\beta_s \cos\Theta} \tag{64}$$

We may further simplify (64) by using the expressions for e_r and $\hat{S}$ in (15) and (13) respectively.

$$\Rightarrow e_r \cdot \hat{S} = \cos\theta \cos\theta_s \cos(\phi-\phi_s) + \sin\theta \sin\theta_s = \cos\beta_s \tag{65}$$

$$\Rightarrow \sin^2\beta_s \cos\theta \frac{\cos\rho}{\cos\gamma_s} = \sin^2\beta_s \sin\theta_s + \frac{C(e_\phi e_r \hat{S})}{\cos\gamma_s} - \cos\beta_s(\sin\theta - \sin\theta_s \cos\beta_s)$$

$$= \sin^2\beta_s \sin\theta_s + \sin\theta_s \cos^2\beta_s + \frac{C(e_\phi e_r \hat{S})}{\cos\gamma_s} - \cos\beta_s \sin\theta$$

$$\cos\rho = \frac{\cos\gamma_s \;[\sin\theta_s - \cos\beta_s \sin\theta] + C(e_\phi e_r \hat{S})}{\sin^2\beta_s \cos\theta} \tag{66}$$

Due to the term $(e_\phi e_r \hat{S})$ an ambiguity results for the angle ρ. As can be seen by figure 22 in this chapter the angle γ_s is a minimum when e_r, $\hat{S}$, and e_ϕ all lie in the same plane. We know from a previous discussion that $(e_\phi e_r \hat{S})$ is positive when e_ϕ is on the same side of the e_r-$\hat{S}$ as $e_\lambda x\hat{S}$. Therefore we may also say that $(e_\phi e_r \hat{S})$ is negative as γ_s goes from its max value to its min value.

We can obtain another expression for the angle ρ by considering the scalar product of (22) with the sun vector $\hat{S}$ expressed in (13).

If

$$A_1 = \cos\theta \sin\theta_s - \cos\theta_s \sin\theta \cos(\phi-\phi_s)$$

$$A_2 = \cos\theta_s \sin(\phi-\phi_s)$$

Then

$$e_\phi \cdot \hat{S} = \cos\gamma_s = A_1 \cos\rho - A_2 \sin\rho$$

$$\Rightarrow A_1^2 \cos^2\rho - 2A_1A_2 \sin\rho \cos\rho + A_2^2 \sin^2\rho = \cos^2\gamma_s$$

$$\Rightarrow A_1^2 + A_2^2 \tan^2\rho - 2A_1A_2 \tan\rho - \cos^2\gamma_s (1+\tan^2\rho) = 0$$

$$\Rightarrow (A_2^2 - \cos^2\gamma_s) \tan^2\rho - 2A_1A_2 \tan\rho + A_1^2 - \cos^2\gamma_s = 0$$

$$\Rightarrow \tan\rho = \frac{A_1A_2 \pm \sqrt{A_2^2 \cos^2\gamma_s + \cos^2\gamma_s(A_1^2 - \cos^2\gamma_s)}}{A_2^2 - \cos^2\gamma_s}$$

$$\tan\rho = \frac{A_1A_2 \pm \cos\gamma_s \sqrt{A_1^2 + A_2^2 - \cos^2\gamma_s}}{A_2^2 - \cos^2\gamma_s} \tag{67}$$

but

$$A_1^2 + A_2^2 = \cos^2\theta \sin^2\theta_s - 2\cos\theta \sin\theta_s \cos\theta_s \sin\theta \cos(\phi-\phi_s)$$

$$+\cos^2\theta_s \sin^2\theta \cos^2(\phi-\phi_s) + \cos^2\theta_s \sin^2(\phi-\phi_s)$$

$$= (1-\sin^2\theta) \sin^2\theta_s - 2 \sin\theta \cos\theta \sin\theta_s \cos\theta_s \cos(\phi-\phi_s)$$

$$+ \cos^2\theta_s (1-\cos^2\theta) \cos^2(\phi-\phi_s) + \cos^2\theta_s \sin^2(\phi-\phi_s)$$

$$= - \sin^2\theta \sin^2\theta_s - 2\sin\theta \cos\theta \sin\theta_s \cos\theta_s \cos(\phi-\phi_s)$$

$$- \cos^2\theta_s \cos^2\theta \cos^2(\phi-\phi_s) + 1$$

$$=1- (\sin\theta \sin\theta_s + \cos\theta \cos\theta_s \cos(\phi-\phi_s))^2$$

so by (65), $A_1^2 + A_2^2 = 1-\cos^2\beta_s = \sin^2\beta_s$ (68)

Inserting (68) into (67) we now have another expression for angle ρ, that is

$$\tan\rho = \frac{A_1A_2 \pm \cos\gamma_s \sqrt{\sin^2\beta_s - \cos^2\gamma_s}}{A_2^2 - \cos^2\gamma_s} \tag{69}$$

The ambiguity can be resolved in the same manner as previously discussed since the term in the square root is exactly that found in (61).*

C. Theoretical description for ($\sphericalangle e_\phi, \hat{U}''$)

Rewriting the expressions for $\hat{U}''$ in (19) and e_ϕ in (22), we have

$$\hat{U}'' = i \cos\Theta_E \cos\{\omega(t-tm) + \phi_s + \phi_E\} + j \cos\Theta_E \sin\{\omega(t-tm) + \phi_s + \phi_E\}$$

$$+ k \sin\Theta_E$$

$$e_\phi = -i(\sin\theta \cos\phi \cos\rho + \sin\phi \sin\rho) - j(\sin\theta \sin\phi \cos\rho - \cos\phi \sin\rho)$$

$$+ k \cos\theta \cos\rho$$

Now taking the scalar product of (19) and (22),

$$e_\phi \cdot \hat{U}'' = - \sin\theta \cos\rho \cos\Theta_E \cos(\phi-[\omega(t-tm) + \phi_s + \phi_E])$$

$$- \sin\rho \cos\Theta_E \sin(\phi-[\omega(t-tm) + \phi_s + \phi_E]) + \cos\theta \cos\rho \sin\Theta_E$$

$$e_\phi \cdot \hat{U}'' = \cos(\sphericalangle e_\phi, \hat{U}) = \cos\Theta_E[\cos\rho\ (\cos\theta \tan\Theta_E - \sin\theta \cos(\phi-[\omega(t-tm) + \phi_s + \phi_E]))$$

$$-\sin\rho \sin(\phi-[\omega(t-tm) + \phi_s + \phi_E])] \tag{70}$$

In summary (70) determines the angle between e_ϕ and U'' where the angle ρ is determined by (69) or (66). When γ_s is increasing, use the + sign for ($e_\phi e_r S$) and when γ_s is decreasing, use the - sign for ($e_\phi e_r S$)

D. Program to determine ($\sphericalangle e_\phi, \hat{U}''$)

In the following program written for the IBM 7094 computer and named ASPECT FINAL, the output is as follows:

* The expression for $\sin\rho$ and its derivation can be found in Appendix F.

ROWP is the angle ρ when $+\sqrt{\sin^2\beta_s - \cos^2\gamma_s}$ was used

ROWN is the angle ρ when $-\sqrt{\sin^2\beta_s - \cos^2\gamma_s}$ was used

ANGLEP is the angle between e_ϕ and $\hat{U}''$ when ROWP was used, and

ANGLEN is the angle between e_ϕ and $\hat{U}''$ when ROWN was used

BANGLP = 360 - ANGLEP

BANGLN = 360 - ANGLEN

SUNPHI = γ_s = the angle between the sun and e_ϕ

GMT = Greenwich mean time for each data point

The only problem that occurred while running this program was at varying times the term

$$\sin^2\beta_s - \cos^2\gamma_s \tag{71}$$

was found to be negative and therefore an error was encountered when determining $(e_\phi e_r \hat{S})$. However this occurred so infrequently that the data points at which (71) was negative were just overlooked.

```
$ID      0232   MARCOTTE    ASPECT FINAL        E3
$TCP            TIME=02,PAGES=15
$ALL            CONTINUE
$IBJOB          DLOGIC
$IBFTC ASPECT   LIST,REF,DECK,SDD
C        ASPECT FINAL
C        DETERMINATION OF ROW FOR ANGLE BETWEEN E-PHI AND U IN FIXED SYSTEM
C        IF SUNPHI IS INCREASING FROM 0 TO 180 DEGREES, USE ROWP
C        IF SUNPHI IS DECREASING FROM 180 TO 0 DEGREES, USE ROWN
       9 READ(5,5)THETAS,PHIS,TM
       5 FORMAT(2F10.5,F10.2)
         THETAS=THETAS*.0174533
         PHIS=PHIS*.0174533
         OMEGA=2.*3.1415927/86164.091
       3 READ(5,1)THETA,PHI,SUNAX,SUNPHI,THETNW,PHINEW,GMT,MTEST
       1 FORMAT (6F7.1,F10.3,4X,I4)
         THETA=THETA*.0174533
         PHI=PHI*.0174533
         SUNAX=SUNAX*.0174533
         SUNPHI=SUNPHI*.0174533
         A=COS(THETA)*SIN(THETAS)-COS(THETAS)*SIN(THETA)*COS(PHI-PHIS)
         B=COS(THETAS)*SIN(PHI-PHIS)
         C=COS(SUNPHI)
         TERM=C*SQRT(SIN(SUNAX)**2-C*C)
         SROWP=A*B+TERM
         SROWN=A*B-TERM
         CROW=B*B-C*C
         HYPOTP=SQRT(SROWP**2+CROW**2)
         HYPOTN=SQRT(SROWN**2+CROW**2)
         SROWP=SROWP/HYPOTP
         SROWN=SROWN/HYPOTN
         CROWP=CROW/HYPOTP
         CROWN=CROW/HYPOTN
C        DETERMINATION OF ANGLE BETWEEN E-PHI AND U
         AMDA=PHINEW*.0174533
         PSI=THETNW*.0174533
         D=COS(THETA)*SIN(PSI)/COS(PSI)-SIN(THETA)*COS(PHI-(OMEGA*
        2(GMT-TM)+PHIS+AMDA))
         E=SIN(PHI-(OMEGA*(GMT-TM)+PHIS+AMDA))
         XCOSP=COS(PSI)*(CROWP*D-SROWP*E)
         XSINP=SQRT(1.-XCOSP*XCOSP)
         ANGLEP=ATAN(XSINP/XCOSP)*57.29578
         XCOSN=COS(PSI)*(CROWN*D-SROWN*E)
         XSINN=SQRT(1.-XCOSN*XCOSN)
         ANGLEN=ATAN(XSINN/XCOSN)*57.29578
         BANGLP=ATAN(-XSINP/XCOSP)*57.29578
         IF(XCOSP)4,3,6
       4 ANGLEP=ANGLEP+180.
         BANGLP=BANGLP+180.
         GO TO 12
       6 BANGLP=BANGLP+360.
      12 BANGLN=ATAN(-XSINN/XCOSN)*57.29578
         IF(XCOSN)7,3,8
       7 ANGLEN=ANGLEN+180.
         BANGLN=BANGLN+180.
         GO TO 13
       8 BANGLN=BANGLN+360.
      13 ROWP=ATAN(SROWP/CROWP)*57.29578
         ROWN=ATAN(SROWN/CROWN)*57.29578
         IF (CROWP) 24,3,26
      24 ROWP=ROWP+180.
```

```
      GO TO 46
   26 IF (SROWP) 44,44,46
   44 ROWP=ROWP+360.
   46 IF (CROWN) 54,3,56
   54 ROWN=ROWN+180.
      GO TO 66
   56 IF (SROWN) 64,64,66
   64 ROWN=ROWN+360.
   66 SUNPHI=SUNPHI*57.29578
      WRITE(6,11)ROWP,ROWN,ANGLEP,BANGLP,ANGLEN,BANGLN,SUNPHI,GMT
   11 FORMAT(1X,8F10.3)
      PUNCH 2,ROWP,ROWN,ANGLEP,BANGLP,ANGLEN,BANGLN,SUNPHI,GMT
    2 FORMAT (8F10.3)
      IF(MTEST-999)3,9,10
   10 CALL EXIT
      STOP
      END
$DATA
```

E. Plots of the Angle between e_ϕ and $\hat{U}''$ with explanations

When plotting ($\sphericalangle e_\phi$, $\hat{U}''$) the rule to choose ANGLEP when γ_s was increasing and ANGLEN when γ_s was decreasing could not be strictly adhered to. The angle γ_s could start increasing or decreasing without switching to the other side of the e_r-$\hat{S}$ plane. In order to determine if the switchover actually occurred, the appropriate angle ρ had to be examined(ROWP for ANGLEP and ROWN for ANGLEN). A switchover from ANGLEP to ANGLEN when γ_s starts decreasing should incur a smooth change from ROWP to ROWN and similarly when γ_s starts increasing.

Another point to bear in mind is that unless ($\sphericalangle\hat{S}$, e_r) is quite small when a critical point occurs for γ_s, then to insure a switchover from one side of the e_r-$\hat{S}$ plane to the other side the angle γ_s should have a min value fairly close to 0° and a max value fairly close to 180°. This can best be seen in figure #22. The vector e_ϕ lies in the $\hat{N}_1$ -$\hat{N}_2$ plane and the min value of γ_s occurs when e_r, $\hat{S}$, and e_ϕ all lie in the same plane.

Still another point to bear in mind is that high detector readings may occur on detectors looking in the direction of sun sensor D even though ($\sphericalangle e_\phi$, $\hat{U}''$) is a large angle. This occurred in revolution 480 approximately 53.5k seconds GMT and the reason was that the detector was looking almost directly into the path of the sun as shown by the plot of ($\sphericalangle S$,e_ϕ) for revolution 480. If the detectors in the direction of the sun sensor D are not looking into the path of the sun and if there are no reflections from the albedo as may have occurred in revolution 957, then these detectors should have high readings when ($\sphericalangle e_\phi$, $\hat{U}''$) has low angular values and low readings when ($\sphericalangle e_\phi$, $\hat{U}''$) has high angular values.

As one can readily see, in the following figures #23-#26, there are points which do not follow the general trend of the curves. These stray points were included in the plots to give a complete picture of the data analyzed. The data listings for these plots can be found in the appendix.

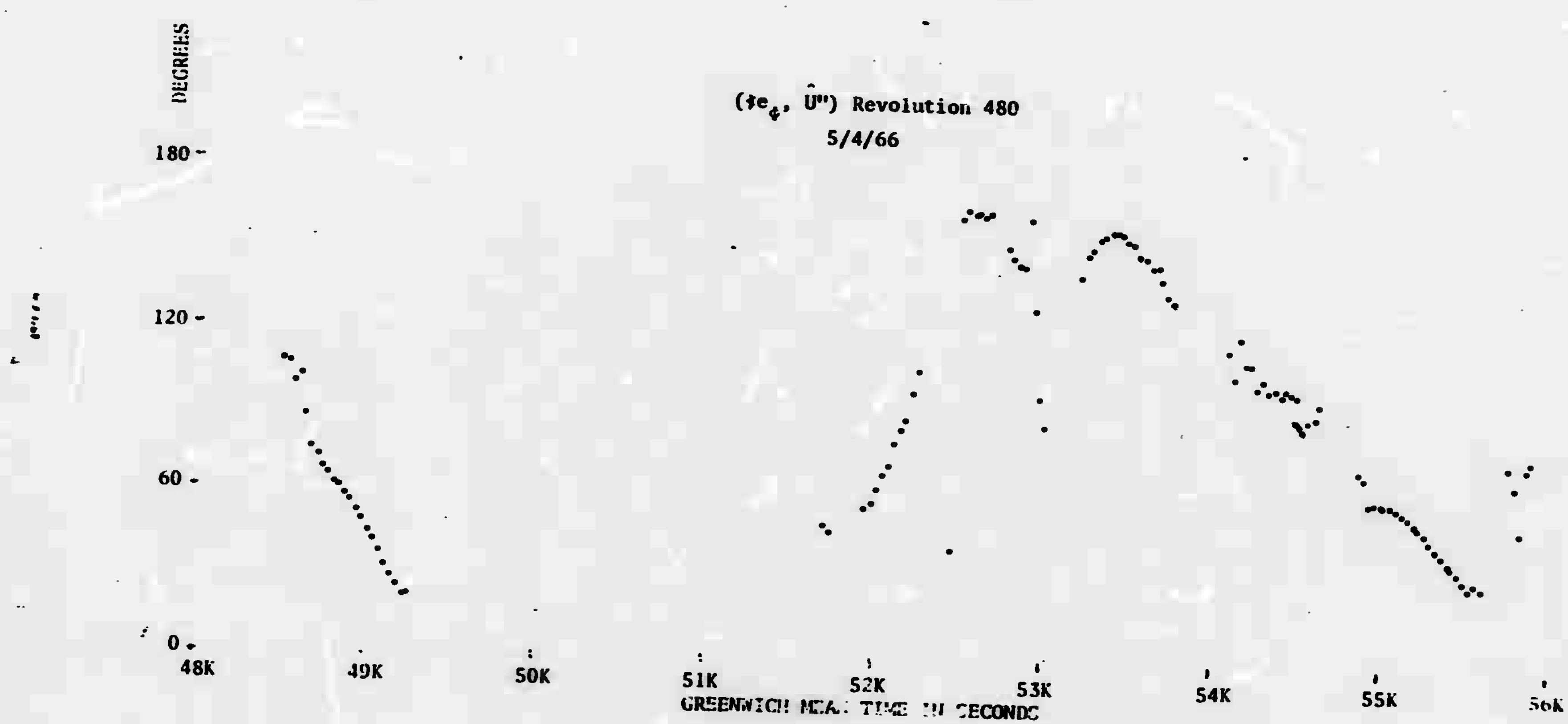
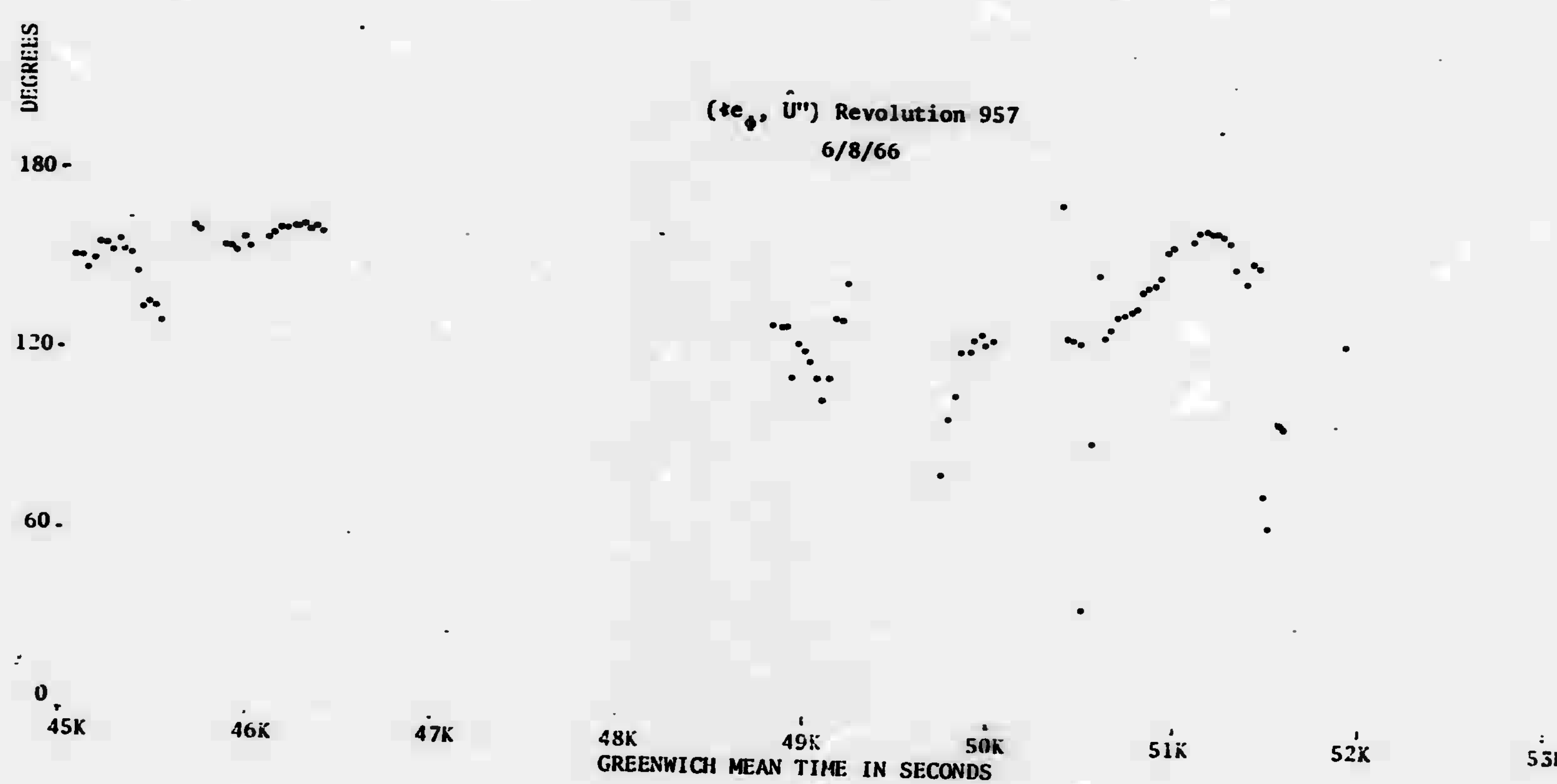

($\neq e_\phi$, $\hat{U}''$) Revolution 480
5/4/66
DEGREES
180
120
60
0
48K
49K
50K
51K
52K
53K
54K
55K
56K
GREENWICH MEAN TIME IN SECONDS

Figure 23



($\star e_\phi$, $\hat{U}''$) Revolution 957
6/8/66
DEGREES
180
120
60
0
45K
46K
47K
48K
49K
50K
51K
52K
53K
GREENWICH MEAN TIME IN SECONDS

Figure 24

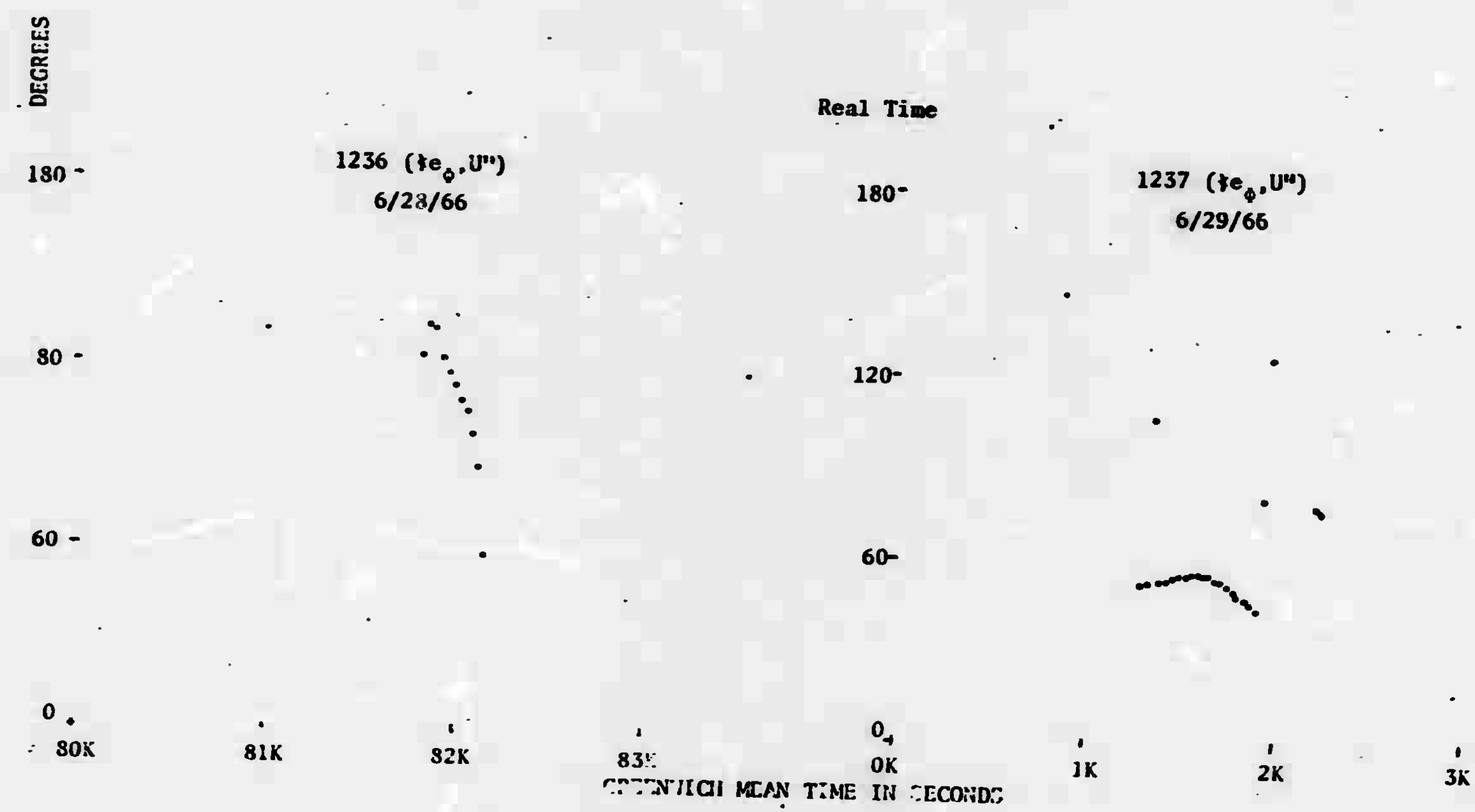

DEGREES
180
80
60
0
80K
81K
82K
83K
1236 ($\nmid e_\phi$,U")
6/28/66
Real Time
180
120
60
0
0K
1K
2K
3K
1237 ($\nmid e_\phi$,U")
6/29/66
GREENWICH MEAN TIME IN SECONDS

Figure 25

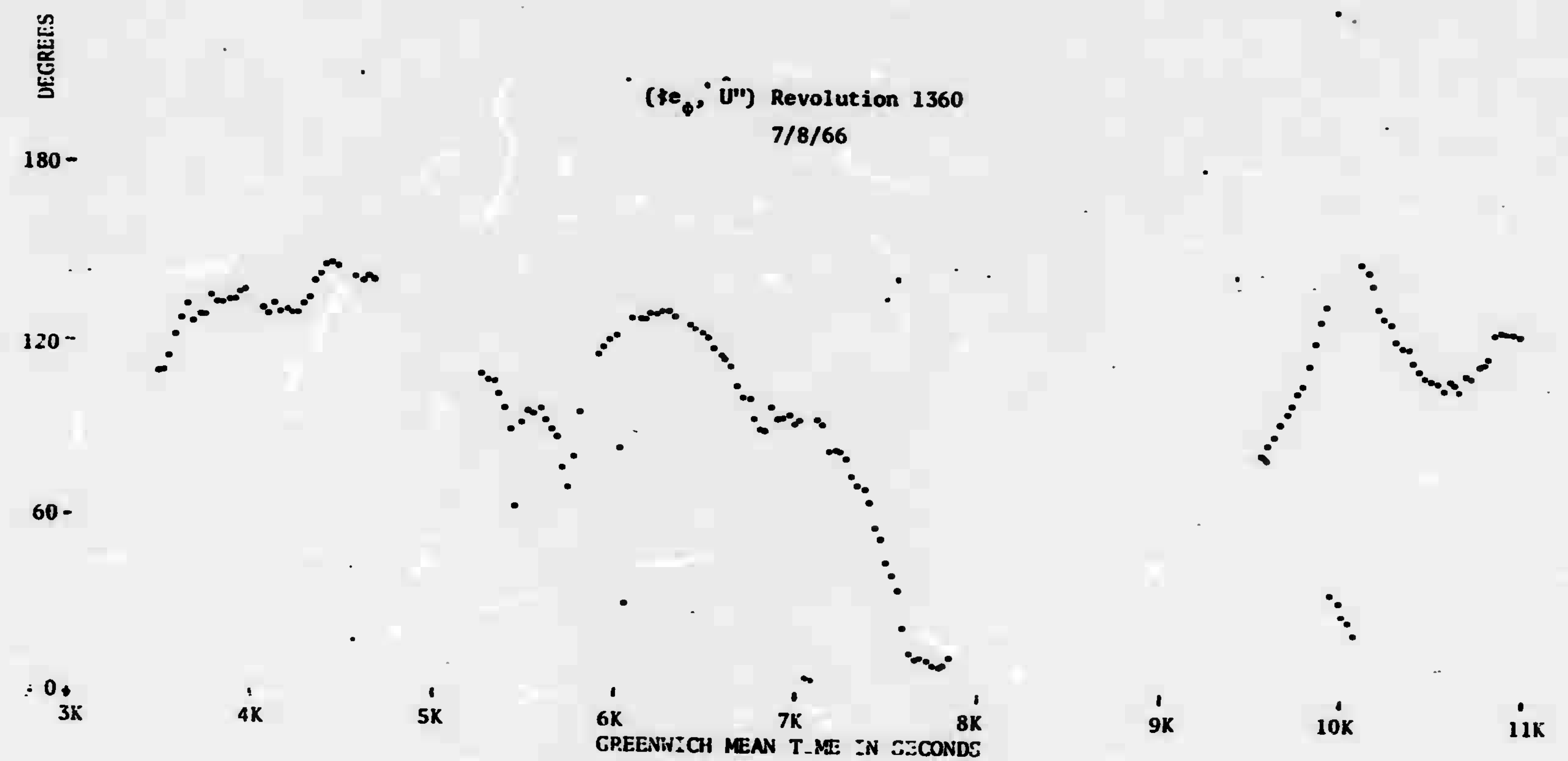

(ê$_\phi$, Û") Revolution 1360
7/8/66
DEGREES
180
120
60
0
3K
4K
5K
6K
7K
8K
9K
10K
11K
GREENWICH MEAN TIME IN SECONDS

Figure 26

APPENDIX

REVOLUTION 480 5/4/66

USE ANGLEP FROM 48519 SECS TO 53414 SECS. AT 53450 SECS SWITCH TO ANGLEN SINCE SUNPHI AND ROWP INDICATE A SWITCH TO THE OTHER SIDE OF THE ER-S PLANE. USE ANGLEN THROUGH 55896 SECS.

ROWP	ROWN	ANGLEP	BANGLP	ANGLEN	BANGLN	SUNPHI	GMT
40.320	46.462	106.174	253.826	111.042	248.958	111.400	48519.000
324.204	328.595	105.137	254.863	102.941	257.059	91.100	48556.000
331.653	342.680	97.954	262.046	91.900	268.100	92.700	48589.000
314.289	65.885	100.401	259.599	45.918	314.082	111.000	48624.000
335.633	45.895	86.115	273.885	46.350	313.650	107.900	48655.000
353.545	25.739	74.067	285.933	53.865	306.135	103.600	48692.000
356.797	23.327	72.131	287.869	55.798	304.202	102.600	48723.000
4.915	14.275	67.002	292.998	61.099	298.901	96.200	48760.000
8.767	12.695	64.559	295.441	62.110	297.890	92.600	48791.000
13.558	11.178	61.598	298.402	63.039	296.961	88.200	48829.000
15.157	12.133	60.306	299.694	62.147	297.853	87.500	48860.000
18.2[illegible]	12.879	57.788	302.212	61.101	298.899	84.500	48897.000
21.475	14.284	55.206	304.794	59.608	300.392	80.200	48928.000
26.605	16.240	51.282	308.718	57.285	302.715	75.900	48965.000
28.989	18.869	48.824	311.176	54.547	305.453	74.300	48996.000
33.705	22.015	44.649	315.351	50.956	309.044	71.200	49034.000
36.372	27.248	41.515	318.485	46.129	313.871	68.500	49065.000
40.050	33.328	37.374	322.626	40.495	319.505	66.000	49102.000
47.798	40.606	32.643	327.357	34.955	325.045	60.900	49133.000
51.863	48.415	28.396	331.604	29.028	330.972	58.700	4917[illegible].000
56.676	56.611	24.965	335.035	24.964	335.036	56.200	49201.000
63.340	60.531	21.924	338.076	21.076	338.924	54.200	49239.000
70.858	66.169	22.277	337.723	19.601	340.399	51.900	4927[illegible].000
62.632	1.346	50.068	309.932	83.833	276.167	59.400	51711.000
67.986	0.240	47.790	312.210	83.556	276.444	56.200	51748.000
81.266	335.037	56.862	303.138	79.404	280.596	37.000	51953.000
81.566	333.495	57.954	302.046	75.877	284.123	36.300	51984.000
78.875	323.479	63.331	296.669	73.767	286.233	33.300	52021.000
73.898	316.415	68.126	291.874	72.288	287.712	32.800	52052.000
72.926	314.168	71.828	288.172	71.234	288.766	32.300	52090.000
69.886	303.686	79.466	280.534	69.723	290.277	30.200	52121.000
65.535	298.516	85.416	274.584	66.170	293.830	31.400	52156.000
61.746	292.099	89.199	270.801	62.378	297.622	32.100	52189.000
61.724	285.700	98.780	261.220	59.032	300.968	32.700	52226.000
64.213	285.088	106.738	253.262	56.669	303.331	33.800	52257.000
89.188	37.792	41.648	318.352	56.753	303.247	69.700	52462.000
80.993	79.072	162.509	197.491	163.231	196.769	55.700	52500.000
67.747	66.710	165.902	194.098	164.753	195.247	59.700	52531.000
63.654	59.819	164.741	195.259	161.935	198.065	63.000	52568.000
62.573	59.383	164.990	195.010	162.252	197.748	65.300	52599.000
61.770	54.863	163.156	196.844	156.382	203.618	70.900	52636.000
64.296	57.226	164.424	195.576	157.367	202.633	71.700	52667.000
65.814	63.850	152.268	207.732	150.386	209.614	84.100	52772.000
67.723	67.119	148.831	211.169	148.259	211.741	88.600	52803.000
71.720	73.041	146.118	213.882	147.340	212.660	92.900	52840.000
77.355	79.776	145.782	214.218	147.992	212.008	96.300	52871.000
108.822	97.880	162.209	197.791	156.446	203.554	103.500	52907.000
167.037	116.137	130.387	229.613	161.542	198.458	107.700	52938.000
204.416	143.277	97.956	262.044	148.489	211.511	114.100	52975.000
210.676	157.735	87.6[illegible]	272.318	44.[illegible]67	315.733	120.200	53006.000
136.416	104.189	142.754	217.246	174.290	185.710	148.600	5321[illegible].000
[illegible].184	104.482	150.392	209.608	167.[illegible]46	190.054	155.600	53247.[illegible]
12[illegible].436	102.008	152.962	207.038	167.497	192.503	159.700	53276.000
117.[illegible]	[illegible].730	156.373	203.627	163.935	196.065	164.800	53315.000

116.513	105.655	157.338	202.662	163.360	196.640	167.300	53346.000
113.206	105.115	158.714	201.286	161.897	198.103	170.500	53383.000
107.628	107.561	158.975	201.025	158.992	201.008	175.000	53414.000
108.479	104.477	157.796	202.204	158.169	201.831	177.100	53450.000
110.074	101.371	155.591	204.409	155.488	204.512	175.400	53481.000
112.035	98.911	155.540	204.460	154.686	205.314	172.700	53518.000
117.471	93.595	154.386	205.614	150.310	209.690	166.700	53549.000
117.653	93.181	155.055	204.945	149.768	210.232	165.600	53586.000
304.572	89.602	28.034	331.966	147.581	212.419	162.100	53617.000
302.413	83.705	30.438	329.562	146.923	213.077	158.300	53654.000
307.364	86.095	25.708	334.292	141.730	218.270	155.600	53685.000
314.390	81.545	27.689	332.311	136.320	223.680	149.500	53722.000
316.250	75.101	30.909	329.091	134.398	225.602	146.900	53753.000
285.851	58.568	62.395	297.605	116.737	243.263	125.100	54092.000
270.439	88.113	73.986	286.014	106.827	253.173	121.300	54128.000
285.108	23.478	74.356	285.644	121.835	238.165	119.200	54159.000
298.698	23.639	74.048	285.952	111.962	248.038	116.100	54196.000
307.340	18.536	75.532	284.468	111.842	248.158	115.100	54227.000
322.987	27.716	70.316	289.684	103.852	256.148	114.100	54263.000
322.132	24.647	74.032	285.968	105.918	254.082	113.700	54294.000
331.992	24.247	74.609	285.391	102.542	257.458	111.000	54330.000
335.491	25.099	75.617	284.383	102.959	257.041	110.800	54361.000
341.913	25.431	76.607	283.393	101.492	258.508	109.100	54395.000
345.351	26.770	78.174	281.826	102.744	257.256	108.900	54429.000
352.177	25.919	80.754	279.246	101.540	258.460	106.000	54465.000
354.729	27.536	80.284	279.716	100.777	259.223	105.900	54496.000
356.905	29.466	67.710	292.290	88.269	271.731	106.000	54532.000
354.986	27.094	74.106	285.894	91.807	268.193	105.600	54563.000
354.679	21.138	80.459	279.541	92.567	267.433	102.600	54599.000
349.967	17.173	87.237	272.763	97.572	262.428	102.700	54635.000
49.088	318.457	26.675	333.325	73.154	286.846	47.300	54869.000
53.234	322.551	29.073	330.927	71.172	288.828	47.500	54900.000
10.674	14.353	63.542	296.458	62.432	297.568	91.400	54933.000
12.424	13.146	62.850	297.150	62.617	297.383	90.300	54964.000
13.158	13.158	62.144	297.856	62.144	297.856	90.000	54999.000
13.588	13.588	61.682	298.318	61.682	298.318	90.000	55030.000
17.011	11.875	59.631	300.369	61.776	298.224	87.100	55066.000
17.686	12.989	58.744	301.256	60.794	299.206	86.900	55097.000
18.211	14.749	57.599	302.401	59.177	300.823	87.100	55133.000
19.559	15.552	56.057	303.943	58.013	301.987	86.300	55164.000
20.234	17.529	54.750	305.250	56.113	303.887	86.400	55199.000
21.223	19.344	53.098	306.902	54.096	305.904	86.300	55230.000
22.611	21.595	51.026	308.974	51.576	308.424	86.100	55265.000
24.610	23.509	48.760	311.240	49.374	310.626	83.600	55296.000
26.372	25.900	46.153	313.847	46.427	313.573	83.600	55332.000
28.093	27.010	44.031	315.969	44.703	315.297	85.500	55363.000
31.874	28.600	40.406	319.594	42.553	317.447	83.000	55399.000
35.028	30.088	36.677	323.323	39.914	320.086	82.400	55430.000
38.827	31.392	32.753	327.247	37.864	322.136	81.900	55465.000
41.860	33.056	29.266	330.734	35.482	324.518	81.500	55496.000
44.681	35.088	25.634	334.366	32.754	327.246	82.100	55531.000
49.992	34.668	21.891	338.109	34.162	325.838	80.200	55562.000
52.429	36.526	19.430	340.570	33.103	326.897	81.300	55598.000
34.077	2.810	51.835	308.165	77.328	282.672	77.100	55762.000
37.650	8.596	45.515	314.485	69.308	290.692	77.500	55798.000
25.513	337.971	98.874	261.126	53.303	306.697	75.700	55829.000
27.983	355.564	58.229	301.771	76.121	283.879	74.700	55865.000
33.084	355.495	87.062	272.938	79.268	280.732	71.800	55896.000

USE ANGLEP FROM 45055 SECS TO 46376 SECS SINCE ROWP IS SMOOTH IN THIS INTERVAL. USE ANGLEN FROM 48801 SECS TO 50926 SECS SINCE SUNPHI INDICATES A POSSIBLE SWITCHOVER AND THE USE OF ANGLEN IN THIS INTERVAL PRODUCES A SMOOTHER CURVE THAN WOULD ANGLEP. USE ANGLEP FROM 50957 SECS TO 51887 SECS DUE TO SUNPHI SWITCHOVER.

ROWP	ROWN	ANGLEP	BANGLP	ANGLEN	BANGLN	SUNPHI	GMT
94.011	103.799	149.855	210.145	146.218	213.782	26.300	45055.000
94.254	105.854	149.880	210.120	144.727	215.273	24.500	45091.000
95.218	112.208	145.520	214.480	136.490	223.510	25.100	45122.000
95.016	111.594	148.727	211.273	139.256	220.744	21.700	45158.000
92.592	114.398	154.020	205.980	139.311	220.689	20.900	45189.000
93.368	121.448	153.859	206.141	133.537	226.463	19.700	45225.000
94.086	127.139	151.292	208.708	125.059	234.941	18.900	45256.000
96.583	195.089	154.983	205.017	64.269	295.731	18.600	45291.000
94.657	148.485	152.156	207.844	105.241	254.759	18.800	45322.000
98.336	168.347	150.702	209.298	86.531	273.469	17.600	45357.000
107.984	209.664	144.287	215.713	47.383	312.617	17.300	45388.000
118.224	225.313	133.089	226.911	28.972	331.028	21.300	45424.000
115.494	227.281	134.698	225.302	26.027	333.973	23.500	45455.000
116.065	230.462	133.491	226.509	23.428	336.572	25.800	45490.000
120.332	226.280	128.996	231.004	27.524	332.476	26.400	45521.000
61.805	334.629	160.294	199.706	112.038	247.962	47.300	45690.000
61.483	334.945	159.163	200.837	113.972	246.028	47.500	45721.000
46.617	14.160	154.268	205.732	129.262	230.738	59.500	45850.000
42.262	23.693	153.795	206.205	139.794	220.206	62.600	45885.000
39.675	23.008	152.331	207.669	139.997	220.003	66.100	45916.000
40.160	26.681	155.291	204.709	145.588	214.412	68.600	45950.000
37.675	28.745	154.156	205.844	148.017	211.983	70.900	45981.000
36.451	31.993	156.944	203.056	154.312	205.688	78.700	46082.000
36.695	33.305	158.303	201.697	156.425	203.575	81.300	46113.000
36.493	35.705	160.020	199.980	159.600	200.400	87.700	46148.000
36.700	36.700	160.185	199.815	160.185	199.815	90.000	46179.000
37.367	37.684	160.540	199.460	160.658	199.342	91.000	46214.000
37.804	39.354	160.617	199.383	161.033	198.967	95.000	46245.000
39.439	41.871	161.355	198.645	161.702	198.298	97.300	46279.000
38.761	42.920	159.730	200.270	159.988	200.012	99.100	46310.000
42.032	50.460	161.030	198.970	159.496	200.504	104.300	46345.000
42.049	52.179	159.149	200.851	156.567	203.433	106.400	46376.000
190.374	164.145	116.028	243.972	131.355	228.645	120.700	48801.000
195.336	165.071	110.329	249.671	130.678	229.322	118.000	48838.000
198.932	164.959	106.581	253.419	130.796	229.204	116.400	48869.000
210.180	184.811	93.138	266.862	113.694	246.306	112.000	48906.000
196.155	174.305	107.791	252.209	125.061	234.939	115.500	48937.000
191.883	177.925	111.343	248.657	122.619	237.381	115.200	48974.000
198.861	180.549	103.922	256.078	119.126	240.874	111.900	49005.000
197.828	186.884	104.226	255.774	113.512	246.488	110.200	49042.000
193.952	193.717	106.232	253.768	106.429	253.571	106.800	49073.000
185.501	185.254	113.535	246.465	113.740	246.260	105.900	49110.000
171.640	159.215	124.477	235.523	133.527	226.473	105.000	49141.000
159.933	159.682	132.773	227.227	132.952	227.048	102.800	49178.000
147.087	137.119	140.896	219.104	145.795	214.205	101.500	49209.000
107.425	139.613	54.627	305.373	82.490	277.510	74.000	49719.000
112.767	159.356	59.766	300.234	101.334	258.666	73.600	49750.000
127.485	166.070	73.207	286.793	108.263	251.737	72.100	49786.000
135.943	181.005	81.621	278.379	123.206	236.794	70.300	49817.000
143.680	179.826	89.982	270.018	123.349	236.651	66.500	49854.000
145.392	184.148	91.934	268.066	127.576	232.424	65.200	49885.000
160.360	183.368	107.163	252.837	128.749	231.251	59.300	49921.000
158.604	179.566	106.166	253.834	125.461	234.539	58.400	49952.000
155.752	180.553	104.520	255.480	127.000	233.000	56.800	49989.000

2[illegible]5.064	250.162	139.333	220.667	172.414	187.586	45.40[illegible]	5035[illegible].000
117.838	154.744	109.185	250.815	128.693	231.307	35.600	50393.000
112.734	150.087	110.978	249.022	127.870	232.130	34.300	50424.000
109.142	143.828	114.237	245.763	126.732	233.268	32.400	5046[illegible].000
303.853	276.531	11.212	348.788	38.122	321.878	29.600	50491.000
101.714	1[illegible]5.519	86.561	273.439	93.790	266.210	30.900	50527.000
303.726	276.001	143.405	216.595	149.773	210.227	28.700	50558.000
100.938	130.734	126.987	233.013	128.204	231.796	27.100	50593.000
99.990	125.540	134.536	225.464	132.461	227.539	22.300	50624.000
101.355	122.905	139.181	220.819	135.707	224.293	19.300	5066[illegible].000
100.647	122.353	141.484	218.516	136.531	223.469	19.100	506[illegible]1.000
100.282	121.267	143.946	216.054	137.433	222.567	18.800	50727.000
98.877	121.937	147.441	212.559	138.665	221.335	18.400	50758.000
103.391	117.429	151.479	208.521	144.291	215.709	13.800	50793.000
103.860	117.567	153.857	206.143	146.013	213.987	12.700	50824.000
106.873	112.776	150.103	209.897	146.697	213.303	14.400	50860.000
105.397	117.647	158.849	201.151	149.546	210.454	9.900	50891.000
112.678	112.720	157.121	202.879	157.083	202.917	5.300	50926.000
110.128	114.757	159.218	200.782	155.008	204.992	5.600	50757.000
109.704	117.097	161.360	198.640	153.967	206.033	5.300	51059.000
106.723	121.742	164.460	195.540	149.612	210.388	7.700	51090.000
105.040	122.400	164.866	195.134	147.783	212.217	9.000	51125.000
104.847	123.991	164.404	195.596	146.352	213.648	9.700	51156.000
102.848	127.045	164.193	195.807	142.668	217.332	12.100	51192.000
102.214	127.987	163.306	196.694	141.422	218.578	12.900	51223.000
102.533	127.927	161.286	198.714	140.635	219.365	12.700	51259.000
112.606	116.417	152.595	207.405	149.377	210.623	2.200	51290.000
71.830	343.236	147.832	212.168	72.370	287.630	45.800	51356.000
93.172	140.428	154.410	205.590	126.502	233.498	23.700	51392.000
90.059	146.424	153.054	206.946	120.442	239.558	26.600	51425.000
67.516	63.490	77.074	282.926	81.057	278.943	88.100	51456.000
38.530	278.574	67.477	292.523	164.527	195.473	31.400	51487.000
50.092	318.162	127.340	232.660	53.085	306.915	52.600	51887.000

REAL TIME 1236 6/28/66

USE ANGLEN TO AGREE WITH DETECTOR READINGS.

ROWP	ROWN	ANGLEP	BANGLP	ANGLEN	BANGLN	SUNPHI	GMT
335.473	339.321	125.886	234.114	122.909	237.091	98.400	81831.000
327.628	333.118	137.332	222.668	133.204	226.796	101.200	81863.000
325.970	332.362	136.714	223.286	131.895	228.105	102.200	81894.000
326.821	337.000	130.775	229.225	122.867	237.133	104.200	81937.000
325.599	339.804	128.909	231.091	117.665	242.335	106.600	81968.000
322.531	342.290	128.948	231.052	113.038	246.962	109.600	82001.000
319.705	346.096	129.580	230.420	107.866	252.134	110.100	82032.000
315.814	349.008	131.951	228.049	104.342	255.658	111.400	82065.000
309.978	356.348	135.913	224.087	97.147	262.853	113.200	82096.000
299.416	6.544	143.096	216.904	86.825	273.175	115.600	82129.000
286.712	37.376	150.624	209.376	58.217	301.783	117.400	82160.000

REAL TIME 1237 6/29/66

USE ANGLEN TO AGREE WITH DETECTOR READINGS AND TO PRODUCE A SMOOTH CHANGE FROM REAL TIME 1236

ROWP	ROWN	ANGLEP	BANGLP	ANGLEN	BANGLN	SUNPHI	GMT
49.248	358.383	9.405	350.595	51.769	308.231	64.600	1309.000
50.898	356.980	8.106	351.894	52.443	307.557	63.100	1340.000
268.543	22.261	59.075	300.925	105.339	254.661	135.800	1373.000
53.492	356.196	4.847	355.153	52.599	307.401	61.500	1406.000
54.993	356.230	7.071	352.929	53.543	306.457	60.900	1446.000
56.243	355.123	8.710	351.290	54.280	305.720	59.800	1477.000
57.841	354.381	10.682	349.318	55.142	304.858	58.700	1509.000
58.145	354.096	11.574	348.426	54.973	305.027	58.500	1540.000
57.820	352.136	12.665	347.335	55.516	304.484	57.900	1571.000
59.333	352.187	14.051	345.949	55.613	304.387	57.200	1602.000
57.920	350.917	14.377	345.623	55.185	304.815	57.500	1634.000
56.717	348.356	15.340	344.660	55.157	304.843	57.100	1665.000
55.862	347.985	15.637	344.363	53.868	306.132	57.400	1697.000
54.824	345.499	16.936	343.064	53.334	306.666	56.900	1728.000
53.163	342.748	18.362	341.638	52.383	307.617	56.600	1760.000
54.008	344.078	19.324	340.676	50.713	309.287	56.700	1791.000
51.223	342.310	20.112	339.888	48.803	311.197	57.400	1823.000
51.787	342.990	21.187	338.813	47.625	312.375	57.400	1854.000
49.827	340.853	23.299	336.701	46.023	313.977	57.500	1886.000
50.905	341.906	25.978	334.022	44.321	315.679	57.300	1917.000
110.428	203.588	164.951	195.049	80.251	279.749	65.100	1959.000
157.848	247.060	63.832	296.168	126.221	233.779	48.800	1988.000

REVOLUTION 1360 7/8/66

USE ANGLEP THROUGHOUT SINCE SUNPHI DID NOT INDICATE A SWITCHOVER AND ROWN IS NOT SMOOTH AT THE BREAK FROM 7846 SECS TO 9595 SECS.

ROWP	ROWN	ANGLEP	BANGLP	ANGLEN	BANGLN	SUNPHI	GMT
55.300	355.409	108.717	251.283	135.187	224.813	56.200	3495.736
49.489	349.177	109.237	250.763	139.057	220.943	58.000	3526.736
49.786	350.793	114.063	245.937	143.744	216.256	58.400	3559.880
46.720	353.652	121.157	238.843	150.093	209.907	61.700	3590.880
43.577	356.957	126.992	233.008	153.153	206.847	65.200	3624.231
40.578	356.624	131.720	228.280	158.051	201.949	67.000	3655.231
21.864	328.684	126.372	233.628	162.540	197.460	65.400	3688.420
19.359	330.816	127.907	232.093	160.892	199.108	67.700	3719.420
19.607	333.486	128.624	231.376	158.849	201.151	68.800	3752.598
14.666	336.080	134.648	225.352	160.501	199.499	72.300	3783.598
14.904	337.731	132.887	227.113	156.836	203.164	73.100	3816.487
15.401	341.137	132.702	227.298	153.656	206.344	74.400	3847.487
13.713	342.684	133.343	226.657	152.041	207.959	76.000	3880.315
13.542	348.539	133.739	226.261	147.834	212.166	78.600	3911.315
10.147	352.806	136.601	223.399	146.050	213.950	82.100	3944.047
9.293	352.405	136.949	223.051	146.046	213.954	82.400	3975.047
23.818	20.835	131.358	228.642	130.489	229.511	88.500	4072.129
20.266	22.284	129.002	230.998	129.653	230.347	91.000	4103.129
23.501	21.255	132.482	227.518	131.578	228.422	88.900	4136.129
19.115	23.883	129.898	230.102	131.982	228.018	92.300	4167.129
16.952	26.474	130.354	229.646	134.794	225.206	94.600	4199.917
16.115	25.954	129.657	230.343	134.550	225.450	94.700	4230.917
14.430	27.663	129.627	230.373	136.635	223.365	96.300	4263.637
16.474	26.959	132.623	227.377	138.600	221.400	95.000	4294.637
14.340	30.912	134.837	225.163	144.147	215.853	98.200	4327.214
14.851	34.065	139.940	220.060	151.121	208.879	99.900	4358.214
14.277	36.436	142.586	217.414	156.070	203.930	101.700	4390.783
15.900	37.549	146.285	213.715	160.897	199.103	101.800	4421.783
16.066	36.446	146.326	213.674	160.939	199.061	101.100	4454.144
14.522	37.801	145.322	214.678	162.680	197.320	102.900	4485.144
10.766	40.672	141.931	218.069	166.790	193.210	107.100	4580.822
9.409	40.660	140.875	219.125	167.361	192.639	107.900	4611.822
10.864	40.110	141.921	218.079	168.410	191.590	107.200	4644.011
10.055	41.026	141.005	218.995	169.905	190.095	108.600	4675.011
349.291	30.164	108.737	251.263	145.691	214.309	112.400	5276.108
347.192	28.944	107.396	252.604	145.720	214.280	112.600	5307.108
345.410	25.442	103.703	253.297	144.154	215.846	110.700	5339.501
339.206	24.004	102.673	257.327	145.414	214.586	111.600	5370.501
330.325	20.194	98.121	261.879	146.963	213.037	111.700	5402.899
313.487	14.272	90.276	269.724	151.022	208.978	110.500	5433.899
258.134	112.583	63.954	296.046	82.302	277.698	110.800	5466.469
35.427	78.786	93.005	266.992	108.988	251.012	110.500	5497.469
17.553	56.145	97.007	262.993	119.182	240.818	110.400	5530.140
10.032	47.837	95.983	264.017	121.147	238.853	110.000	5561.140
7.873	38.008	92.141	267.859	119.223	240.777	106.200	5593.527
3.274	36.809	93.465	266.535	118.920	241.080	107.700	5624.527
1.129	33.476	91.305	268.695	116.637	243.363	106.900	5656.987
.177	31.099	87.490	272.510	112.057	247.943	105.700	5687.987
355.882	32.184	77.575	282.424	106.919	253.081	107.900	5720.494
355.613	28.032	71.207	288.793	98.123	261.877	105.600	5751.494
358.905	26.150	80.518	279.482	102.620	257.380	103.300	5783.913
359.365	26.398	96.846	263.154	119.118	240.882	103.300	5814.913
4.[illegible]80	29.577	116.727	243.273	139.358	220.642	102.700	5910.951
6.424	28.118	119.261	240.739	138.662	221.338	100.800	5941.951
4.474	31.078	121.573	238.427	145.844	214.156	103.300	5974.351

6.401	30.212	123.018	236.982	144.735	215.265	101.900	6005.351
351.579	329.605	84.201	275.799	63.601	296.399	70.300	6031.434
317.830	29.482	32.736	227.264	95.216	264.784	125.600	6062.434
9.170	26.562	128.843	231.157	144.410	215.590	98.700	6100.840
7.365	27.010	128.684	231.316	145.956	214.044	99.800	6131.840
6.249	26.334	128.428	231.572	145.313	214.687	100.000	6163.959
7.947	23.241	130.626	229.374	143.069	216.931	97.600	6194.959
5.519	22.511	130.213	229.787	143.258	216.742	98.400	6227.278
5.797	21.989	131.283	228.717	143.531	216.469	98.000	6258.278
6.738	20.912	131.123	228.877	141.184	218.816	97.000	6290.704
8.289	19.417	129.807	230.193	137.006	222.994	95.500	6321.704
2.493	18.564	126.598	233.402	134.102	225.898	97.800	6417.802
6.013	17.478	125.002	234.998	130.024	229.976	95.600	6448.802
5.819	16.511	124.047	235.953	128.176	231.824	95.200	6481.456
17.287	17.727	122.250	237.750	122.482	237.518	93.900	6512.456
17.655	17.723	118.783	241.217	118.818	241.182	91.000	6545.017
18.185	18.323	117.075	242.925	117.145	242.855	92.900	6576.017
18.006	18.117	114.414	245.586	114.468	245.532	91.500	6608.429
18.267	18.341	112.245	247.755	112.280	247.720	90.700	6639.429
18.110	18.269	106.142	253.858	106.220	253.780	92.200	6671.882
17.817	17.873	101.832	258.168	101.860	258.140	90.600	6702.882
17.898	17.951	101.470	258.530	101.495	258.505	90.600	6735.263
17.120	17.049	94.412	265.588	94.376	265.624	88.700	6766.263
343.335	343.322	91.069	268.931	91.075	268.925	89.400	6798.633
343.127	343.196	90.370	269.630	90.336	269.664	90.600	6829.633
341.142	340.985	98.594	261.406	98.661	261.339	87.900	6861.979
19.826	14.268	94.551	265.449	92.960	267.040	87.200	6892.979
19.120	14.758	95.083	264.917	94.050	265.950	87.800	6925.404
20.476	12.730	95.789	264.211	94.381	265.619	86.100	6956.404
19.533	13.974	92.817	267.183	91.828	268.172	87.200	6988.762
19.603	12.412	94.404	265.596	93.715	266.285	86.400	7019.762
289.526	286.699	6.802	353.198	8.795	351.205	88.900	7054.255
290.192	290.944	6.918	353.082	6.430	353.570	90.300	7085.255
15.633	13.192	94.901	265.099	95.019	264.981	88.800	7115.498
16.808	10.633	92.833	267.167	93.451	266.549	87.000	7146.498
18.235	11.283	83.791	276.209	84.488	275.512	86.600	7179.127
18.633	7.198	84.087	275.913	86.511	273.489	84.600	7210.127
16.224	8.515	83.473	276.527	85.461	274.539	86.400	7242.829
18.546	5.706	80.765	279.235	84.549	275.451	84.000	7273.829
17.270	8.389	75.511	284.489	78.458	281.542	85.900	7306.578
18.941	7.000	72.557	287.443	76.853	283.147	84.500	7337.578
18.968	5.984	71.227	288.773	76.476	283.524	84.100	7370.309
18.157	10.254	66.500	293.500	69.556	290.444	86.300	7401.309
24.207	5.335	58.176	301.824	67.229	292.771	81.400	7434.103
24.223	7.973	54.022	305.978	61.809	298.191	82.400	7465.103
25.769	9.924	47.279	312.721	55.187	304.813	82.400	7497.938
26.093	11.935	42.991	317.009	50.902	309.098	83.200	7528.938
26.075	14.282	37.753	322.247	44.238	315.762	84.100	7561.781
30.542	15.989	24.504	335.496	33.651	326.349	82.300	7592.781
32.215	16.060	15.755	344.245	27.787	332.213	81.100	7625.497
32.257	17.884	14.024	345.976	26.182	333.818	[illegible]	7656.497
29.803	17.884	14.854	345.146	25.369	334.631	83.300	7688.930
30.201	17.806	13.645	346.355	25.385	334.615	82.900	7719.930
32.403	18.015	12.192	347.808	26.507	333.493	81.400	7752.193
33.956	17.085	11.644	348.356	28.419	331.581	79.500	7783.193
35.572	16.260	12.224	347.776	30.911	329.089	77.600	7815.474
35.063	16.526	15.207	344.793	32.398	327.602	[illegible]	7846.474
30.979	353.761	87.327	272.673	111.981	248.019	72.100	9595.632
28.015	354.115	[illegible]	269.781	113.360	246.640	[illegible]	9626.632
24.839	354.127	94.370	265.630	115.940	244.060	[illegible]	9659.455
[illegible]	354.592	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

21.272	354.150	101.194	258.806	120.464	239.536	77.700	9723.337
21.153	355.274	104.344	255.056	122.831	237.169	78.100	9754.337
17.623	353.762	107.147	251.853	125.327	234.673	79.400	9787.166
12.608	353.689	114.243	245.757	128.337	231.663	81.800	9818.166
6.408	357.887	121.647	238.153	131.466	228.514	84.700	9850.665
.309	351.786	128.830	231.170	135.670	224.330	86.500	9881.865
356.310	351.904	134.868	225.132	138.447	221.553	88.200	9914.632
331.050	329.850	36.416	323.084	36.283	323.717	89.400	9945.632
329.148	330.948	35.319	324.681	36.204	323.796	90.900	9978.266
321.923	329.524	[illegible]	329.550	34.261	325.739	93.800	10009.266
317.950	327.754	27.720	332.280	32.489	327.511	94.900	10041.581
312.026	324.643	23.929	336.071	29.966	330.034	96.300	10072.681
350.050	7.823	149.080	210.020	137.781	222.219	97.600	10105.413
350.073	12.423	147.578	212.422	133.438	226.562	99.600	10136.413
352.985	20.665	141.846	213.154	127.423	232.577	102.500	10169.113
.252	23.635	134.154	225.846	124.899	235.101	101.200	10200.113
1.098	27.150	130.525	229.475	122.612	237.388	102.800	10232.928
.704	28.992	128.819	231.181	121.715	238.285	104.000	10263.928
.521	31.384	123.605	236.395	118.591	241.409	105.400	10296.777
358.969	34.155	121.080	238.920	117.728	242.272	107.700	10327.777
358.327	34.739	120.851	239.149	118.347	241.653	108.300	10360.833
359.884	34.150	115.960	244.040	117.547	242.453	107.200	10391.833
357.687	36.120	112.572	247.428	117.614	242.386	109.100	10424.668
355.824	37.446	110.999	249.001	117.690	242.310	110.600	10455.568
355.276	37.639	109.905	250.195	119.682	240.318	110.700	10488.521
355.587	36.753	109.180	250.820	120.885	239.115	109.900	10519.521
352.410	39.208	107.290	252.710	121.079	238.921	112.600	10552.547
353.134	38.847	109.035	250.965	122.918	237.082	112.200	10583.547
351.066	41.019	108.573	251.427	126.585	233.415	114.000	10616.650
348.397	42.649	105.994	254.006	127.282	232.718	115.800	10647.650
351.434	41.698	111.228	248.772	132.081	227.919	114.200	10680.718
351.231	41.595	110.856	249.144	134.104	225.896	114.000	10711.718
350.981	43.201	113.503	246.497	137.061	222.939	115.400	10744.865
350.738	43.791	115.006	244.994	138.739	221.261	116.100	10775.865
351.325	44.929	118.160	241.840	143.560	216.440	116.700	10809.045
352.414	45.074	125.777	234.223	149.439	210.561	117.000	10840.045
354.069	44.218	126.673	233.327	152.183	207.817	115.800	10873.140
354.210	44.290	126.456	233.544	153.895	206.105	115.800	10904.140
354.024	43.897	125.196	233.804	154.849	205.151	115.700	10937.273
353.537	44.416	125.422	234.578	156.254	203.746	116.200	10968.273

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$ID     0232    MARCOTTE    EGHEVA             E3
$TCP            TIME=02,PAGES=15
$ALL            CONTINUE
$IBJOB          DLOGIC
$IBFTC EGHEVA   LIST,REF,DECK,SDD
CEGHEVA
C
C     PROGRAM SHUVE, NEEDS FCTRAL, FIELD
C     EPOCH CORRECTED FIELD VS. ALT.
      DIMENSION A(1000),G(30,30),H(30,30),FMT(12)
     1,TG(30,30),TH(30,30),GCORR(30,30),HCORR(30,30)
      COMMON  HCORR  , GCORR
C     KTIME =3, GET POSITION
C     TIME = TIME DIFFERENCES IN YEARS
C     KTIME=1,2 COEFFS ARE FOR MAIN FIELD, RATE OF CHANGE
      LOUNT=50
      PI=3.14159265
      PIDEG=5729.57795E-2
1     READ (5,9000)KPROG,NDUMMY,COMULT,KTH ,KTIME,TIME
9000  FORMAT(2I4,E16.8,2I4,E16.8)
      KOUNT=0
      R2=0.0
      GO TO (12,12,2010),KTIME
C
C     NDUMMY IS NO. OF COEFFICIENTS.
C     COMULT IS A MULTUPLIER FOR COEFFS.
C     KPROG IS 1,  A IS NOT MODIFIED.      KPROG =2,  A=A*COMULT.
C     KPROG = 3  FOR SCHMIDT A.  KPROG = 4  FOR VESTINE TYPE (N*SCHMIDT)
C
12    DO 112 K=1,1000
112   A(K)=0.0
C
C     COMPUTE N1,M22,M1, FOR FBASIC;  NTOP IS NO. OF TERMS IN COMPLETE
C     SET OF HIGHEST DEGREE.  NEXTRA IS NO. OF TERMS IN INCOMPLETE SET.
C
      Q000FL=SQRT(FLOAT(NDUMMY+1)+.01)
      NPART=Q000FL+1.0
C
C     READ IN COEFFS.  A10,A11,B11,A20........
C
9     READ (5,100)(FMT(I),I=1,12)
100   FORMAT(12A6)
      READ (5,FMT)(A(J),J=1,NDUMMY)
C     FIND MODIFIED COEFFS.
C
      GO TO(17,92,91,91),KPROG
92     DO93 LL=1,NDUMMY
93     A(LL)=A(LL)*COMULT
      GO TO 17
91    I=0
      N=0
      AA=2.0*(SQRT(FLOAT(NDUMMY))+1.0)+1.0
       KK=AA
        DO10000 K=3,KK,2
       N=N+1
       DO10000 J=1,K
       I=I+1
      M = (I - N**2 +1)/2
      IF(M) 1002,15,16
   15 FACTOR =FCTRAL (2*N)/(2.0**N*(FCTRAL(N))**2)
      GO TO 14
```

```
   16 FACTOR =FCTRAL(2*N)/(2.0**N*FCTRAL(N))*SORT(2.0/(FCTRAL(N+M)* FCTR
     1AL(N-M)))
14        GO TO (1007,1010,14003,14002),KPROG
14003      FACTOR =FACTOR * COMULT
          GO TO 14001
14002 FACTOR = FACTOR * COMULT /FLOAT(N)
14001 A(I)=FACTOR *A(I)
         IF(I-NDUMMY)10000,17,1111
10000  CONTINUE
17     IX=0
       DO 2000 NX=2,NPART
       DO 2000 MX=1,NX
       IX=IX+1
       IF(MX-1)2001,2002,2003
2002   GO TO (50,51),KTIME
50     G(NX,1)=A(IX)
       GO TO 2000
51     TG(NX,1)=A(IX)
       GO TO 2000
2003   GO TO (52,53),KTIME
53     TG(NX,MX)=A(IX)
       IX=IX+1
       TH(NX,MX)=A(IX)
       GO TO 2000
52     G(NX,MX)=A(IX)
       IX=IX+1
       H(NX,MX)=A(IX)
2000   CONTINUE
       GO TO 1
2010   DO 55 JJ=1,30
       DO 55 KK=1,30
       HCORR(JJ,KK)=H(JJ,KK)+TIME*TH(JJ,KK)
55     GCORR(JJ,KK)=G(JJ,KK)+TIME*TG(JJ,KK)
12010  READ (5,7000) CIME,AUNCH,THETA,PHI,HEIGHT
 7000  FORMAT (2F10.4,3(2XF8.4))
       HEIGHT=HEIGHT*1.85325
       IF(CIME-99999.0)6001,4002,2001
4002   COUNT=KOUNT
       SSOR=SORT(R2/COUNT)
       WRITE (6,5005)SSOR
5005   FORMAT(E16.8)
       GO TO 22
6001   CALL FIELD(THETA,PHI,HEIGHT,NPART,X,Y,Z,F)
       KOUNT=KOUNT+1
       HELL=SORT(X*X+Y*Y)
       ANC=PIDEG*ATAN(ABS(Z/HELL))
       IF(X)500,501,502
500    D=SIGN(PI,Y)-SIGN(ATAN(Y/X),Y)
       GO TO 503
501    D=SIGN(PI*.5,Y)
       GO TO 503
502    D=ATAN(Y/X)
503    D=PIDEG*D
       IF(LOUNT-50) 1012,1011,1012
 1011  WRITE(6,1013)
       LOUNT=0
 1012  WRITE(6,110) THETA,PHI,HEIGHT,X,Y,Z,HELL,F,D,ANC,AUNCH
       PUNCH 111,THETA,PHI,HEIGHT,X,Y,Z,F,AUNCH
  110  FORMAT(2F9.4,1PE16.8,-2P5F12.3,0P2F12.3,F12.4)
  111  FORMAT(3F10.3,-2P4F10.3,0PF10.3)
       LOUNT=LOUNT+1
```

```
      GO TO 17010
1002   CALL DUMP
1003   CALL DUMP
1007   CALL DUMP
1010   CALL DUMP
1111   CALL DUMP
2001  CALL DUMP
 1013 FORMAT(132H1  E. LAT. N LONG.  HEIGHT KM.      NORTH X       EAST Y
     1     DOWN Z        HORIZ     TOTAL INTENS. DECLINATION INCLINATION T A
     2FTER LAUNCH)
   22 CALL EXIT
      STOP
      END
$IBFTC IFELD   LIST,REF,DECK,SDD
      SUBROUTINE FIELD(DLAT,DLONG,HGT,NMAX,BN,BE,BV,B)
C     EARTHS MAGNETIC FIELD USING ANY SET OF COEFFICIENTS
      DIMENSION H(30,30),G(30,30),P(30,30),DP(30,30),CONST(30,30),SP(30)
     1,CP(30),AOR(30)
      COMMON H,G
      IF(CP(1)-1.0)1,2,1
1     P(1,1)=1.0
      DP(1,1)=0.0
      SP(1)=0.0
      CP(1)=1.0
      DO 4 M=1,30
      DO 3N=1,2
3     CONST(N,M)=0.0
      DO 4 N=3,30
      FM=M
      FN=N
4     CONST(N,M)=((FN-2.0)*(FN-2.0)-(FM-1.0)*(FM-1.0))/((FN+FN)-3.0)/((F
     1N+FN)-5.0)
2     PHI=DLONG/57.2957795
      AR=6371.2/(6371.2+HGT)
      C=SIN (DLAT/57.2957795)
      S=SQRT (1.0-C*C)
      SP(2)=SIN (PHI)
      CP(2)=COS (PHI)
      AOR(1)=   AR*AR
      AOR(2)=    AR*AOR(1)
      DO 5 M=3,NMAX
      SP(M)=SP(2)*CP(M-1)+CP(2)*SP(M-1)
      CP(M)=CP(2)*CP(M-1)-SP(2)*SP(M-1)
5     AOR(M)=    AR*AOR(M-1)
      BV=0.
      BN=0.0
      BPHI=0.0
      DO 6 N=2,NMAX
      FN=N
      SUMR=0.0
      SUMT=0.0
      SUMP=0.0
      DO 7 M=1,N
      IF(N-M)8,9,8
9     P(N,N)=S*P(N-1,N-1)
      DP(N,N)=S*DP(N-1,N-1)+C*P(N-1,N-1)
      GO TO 10
8     P(N,M)=C*P(N-1,M)-CONST(N,M)*P(N-2,M)
      DP(N,M)=C*DP(N-1,M)-S*P(N-1,M)-CONST(N,M)*DP(N-2,M)
10    FM=M-1
      TS=G(N,M)*CP(M)+H(N,M)*SP(M)
```

```
      SUMR=SUMR+P(N,M)*TS
      SUMT=SUMT+DP(N,M)*TS
7     SUMP=SUMP+FM*P(N,M)*(-G(N,M)*SP(M)+H(N,M)*CP(M))
      BV=BV+AOR(N)*FN*SUMR
      BN=BN-AOR(N)*SUMT
6     BPHI=BPHI-AOR(N)*SUMP
      BF=-BPHI/S
      B=SQRT (BN*BN+BV*BV+BF*BF)
      RETURN
      END
$IBFTC CFTRAL  LIST,REF,DECK,SDD
      FUNCTION FCTRAL(K)
    5 IF(K)2000,1,2
 2000 STOP 2000
    1 FCTRAL=1.0
      RETURN
    2 IF(K-1)2000,1,3
    3 PROD=FLOAT(K)
      Q000FL=PROD
    4 Q000FL=Q000FL-1.0
    6 IF(Q000FL-1.0)2001,7,10
 2001 STOP 2001
    7 FCTRAL=PROD
      RETURN
   10 PROD=PROD*Q000FL
      GO TO 4
      END
$DATA
    3  48  -1.8973926E-06    2    2
(24XE25.8)
      1                                               .136E 2
      2                                               .590E 1
      3                                               .230E 1
      4                                              -.215E 2
      5                                              -.180E 1
      6                                              -.160E 2
      7                                              -.100E 0
      8                                              -.171E 2
      9                                               .190E 1
     10                                              -.820E 1
     11                                               .820E 1
     12                                               .110E 1
     13                                               .350E 1
     14                                              -.500E 0
     15                                              -.670E 1
     16                                              -.440E 1
     17                                               .270E 1
     18                                               .000E 0
     19                                              -.200E 1
     20                                              -.110E 1
     21                                              -.100E 1
     22                                               .350E 1
     23                                              -.220E 1
     24                                              -.210E 1
     25                                               .210E 1
     26                                              -.180E 1
     27                                               .180E 1
     28                                               .200E 0
     29                                               .170E 1
     30                                               .100E 1
     31                                              -.380E 1
```

```
      32                                        -.500E 0
      33                                         .700E 0
      34                                        -.200E 0
      35                                         .200E 0
      36                                         .170E 1
      37                                         .210E 1
      38                                        -.700E 0
      39                                         .100E 0
      40                                        -.200E 0
      41                                        -.200E 0
      42                                         .150E 1
      43.                                        .150E 1
      44                                        -.110E 1
      45                                         .500E 0
      46                                         .700E 0
      47                                        -.600E 0
      48                                         .100E 1
   2  77         -1.          1    1
(24XF25.8)
   1                                   -0.30508775E 05
   2                                   -0.21808302E 04
   3                                    0.58407369E 04
   4                                   -0.21956077E 04
   5                                    0.51452178E 04
   6                                   -0.34430141E 04
   7                                    0.14484838E 04
   8                                    0.17150113E 03
   9                                    0.28666562E 04
  10                                   -0.61608247E 04
  11                                   -0.11936724E 04
  12                                    0.22290292E 04
  13                                    0.56309372E 03
  14                                    0.65680435E 03
  15                                   -0.13761476E 03
  16                                    0.43600536E 04
  17                                    0.47754694E 04
  18                                    0.95552545E 03
  19                                    0.22800695E 04
  20                                   -0.11412046E 04
  21                                   -0.65218185E 03
  22                                    0.14499263E 03
  23                                    0.17518126E 03
  24                                   -0.15226956E 03
  25                                   -0.21027692E 04
  26                                    0.27628720E 04
  27                                   -0.39691649E 03
  28                                    0.19293695E 04
  29                                    0.12409761E 04
  30                                   -0.24542910E 03
  31                                   -0.59848446E 03
  32                                   -0.24982365E 03
  33                                   -0.32559201E 03
  34                                   -0.57247381E 01
  35                                    0.61958762E 02
  36                                    0.69252063E 03
  37                                    0.24850067E 04
  38                                    0.12455038E 02
  39                                   -0.22692592E 03
  40                                    0.92501821E 03
  41                                   -0.22338884E 04
  42                                    0.37497479E 03
```

43	0.70218083E 02
44	0.10117546E 03
45	-0.16596319E 03
46	0.87118310E 01
47	-0.44940472E 02
48	-0.38003508E 02
49	0.27806275E 04
50	-0.22568674E 04
51	-0.24383128E 04
52	0.22736780E 03
53	0.19120144E 03
54	-0.37440271E 03
55	0.75827096E 03
56	-0.51570851E 03
57	0.26619910E 03
58	0.33689441E 03
59	0.51373941E 02
60	-0.15688118E 02
61	0.14926150E 03
62	-0.21241840E 02
63	0.17769346E 01
64	-0.81528097E 03
65	0.40691421E 03
66	-0.37214176E 03
67	0.72929968E 03
68	-0.11764831E 04
69	-0.19003021E 03
70	-0.42363667E 02
71	0.88395876E 03
72	-0.18684875E 04
73	-0.15495010E 03
74	0.80660194E 02
75	-0.47169191E 02
76	-0.75128689E 02
77	0.15357842E 03
3	6.50000000E+00

Figure 27

LEAST SQUARES APPROXIMATION FOR SUN SENSOR A
OUTPUT VOLTAGE 1

$$\Theta=-45.113-1.273V+14.399V^2-2.167V^3$$

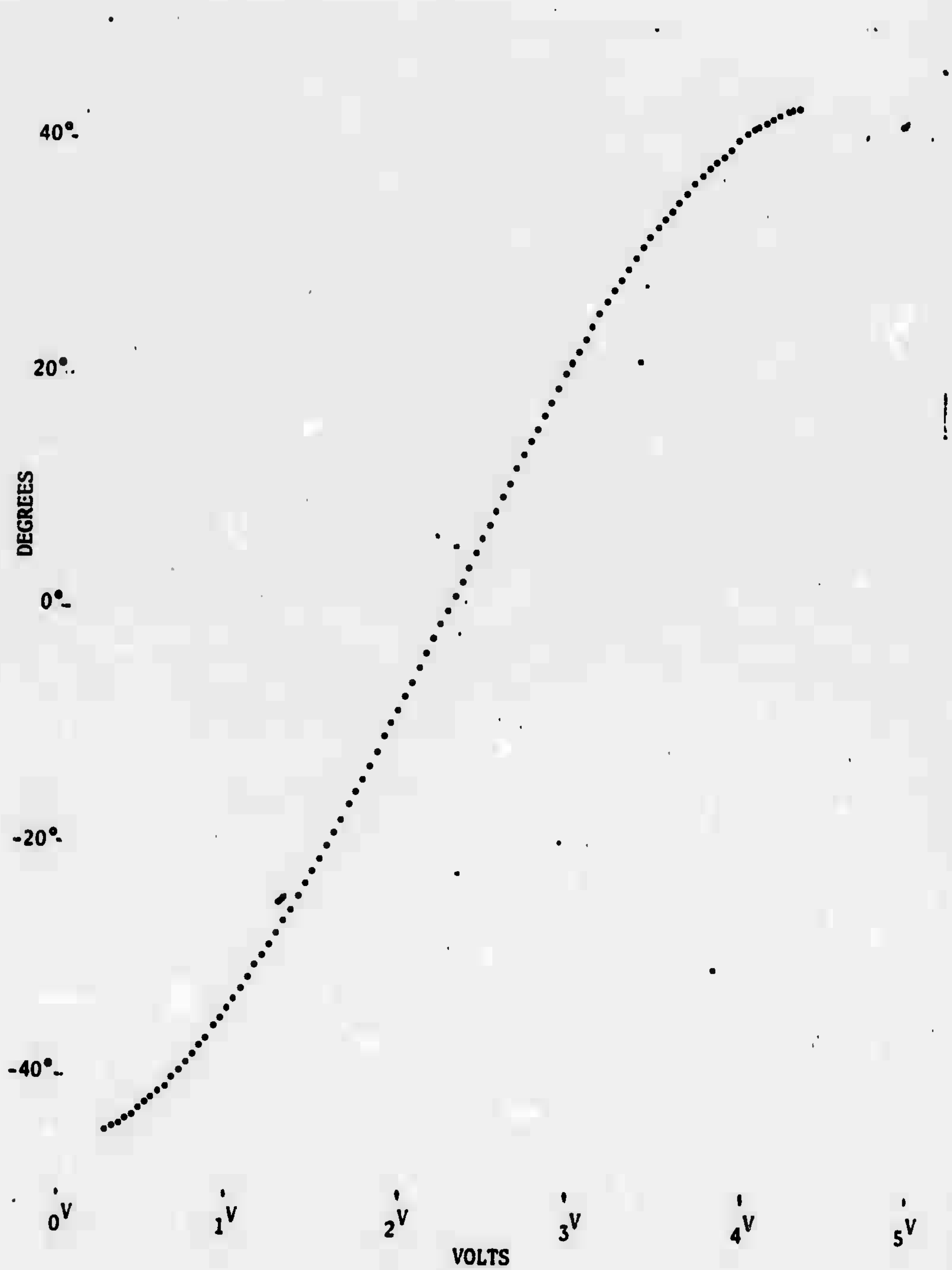


Figure 28

LEAST SQUARES APPROXIMATION FOR SUN SENSOR A
OUTPUT VOLTAGE 2

$$\theta=-44.852+1.363V+13.063V^2-1.984V^3$$

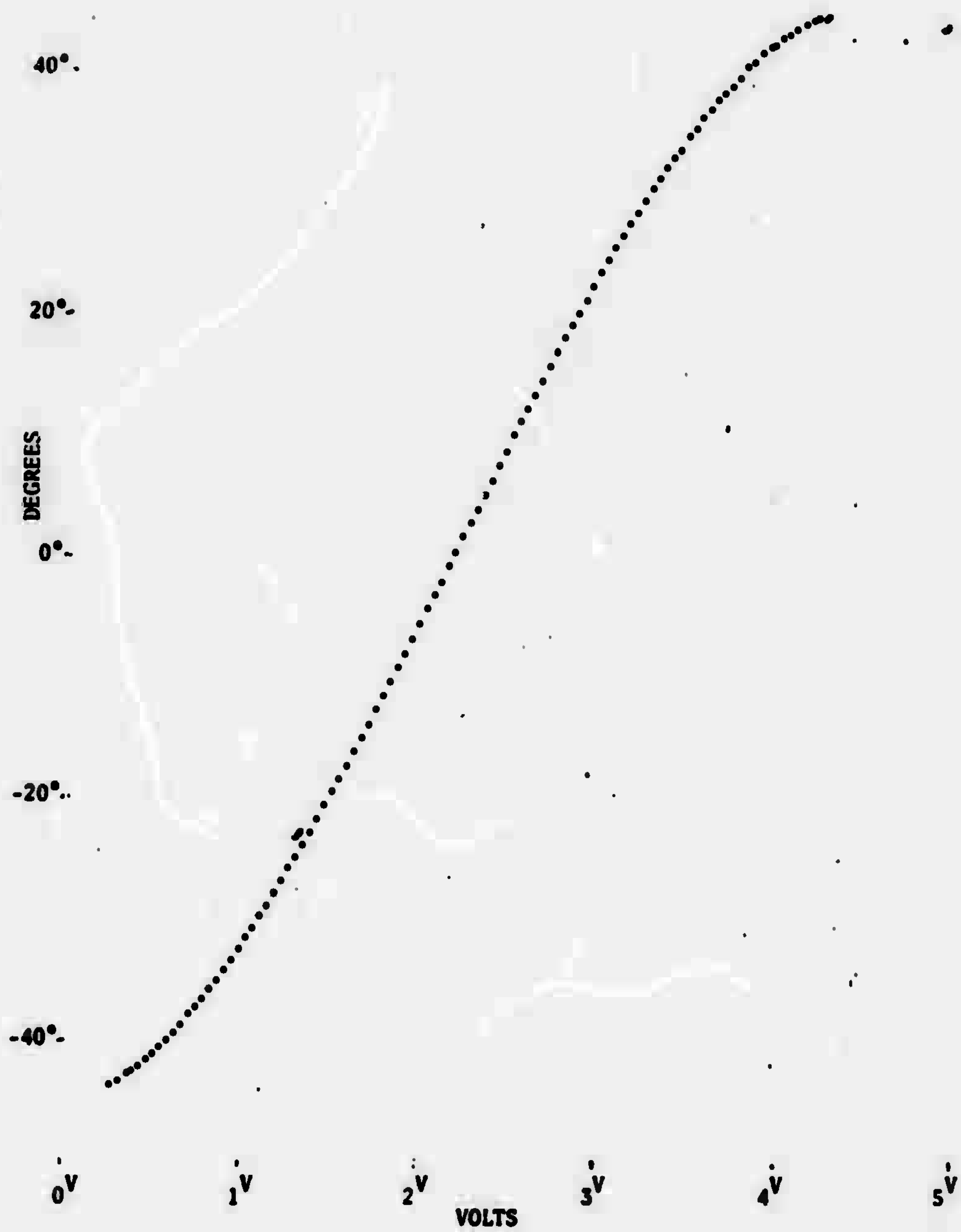


Figure 29

LEAST SQUARES APPROXIMATION FOR SUN SENSOR B
OUTPUT VOLTAGE 1

$\theta = -45.956 + 4.636V + 11.970V^2 - 1.901V^3$

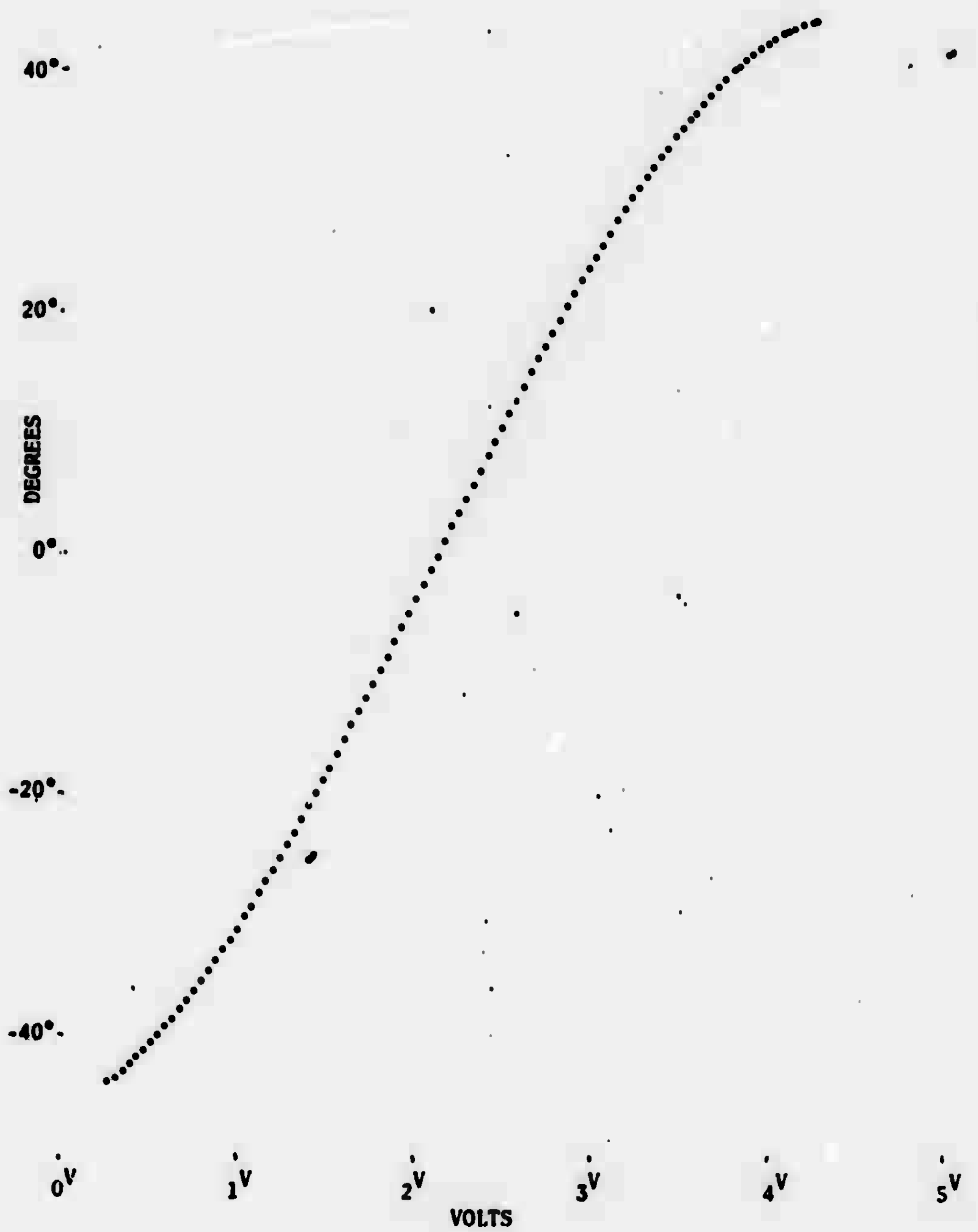


Figure 30

LEAST SQUARES APPROXIMATION FOR SUN SENSOR B
OUTPUT VOLTAGE 2

$\theta = -45.291 + 3.724V + 12.014V^2 - 1.873V^3$

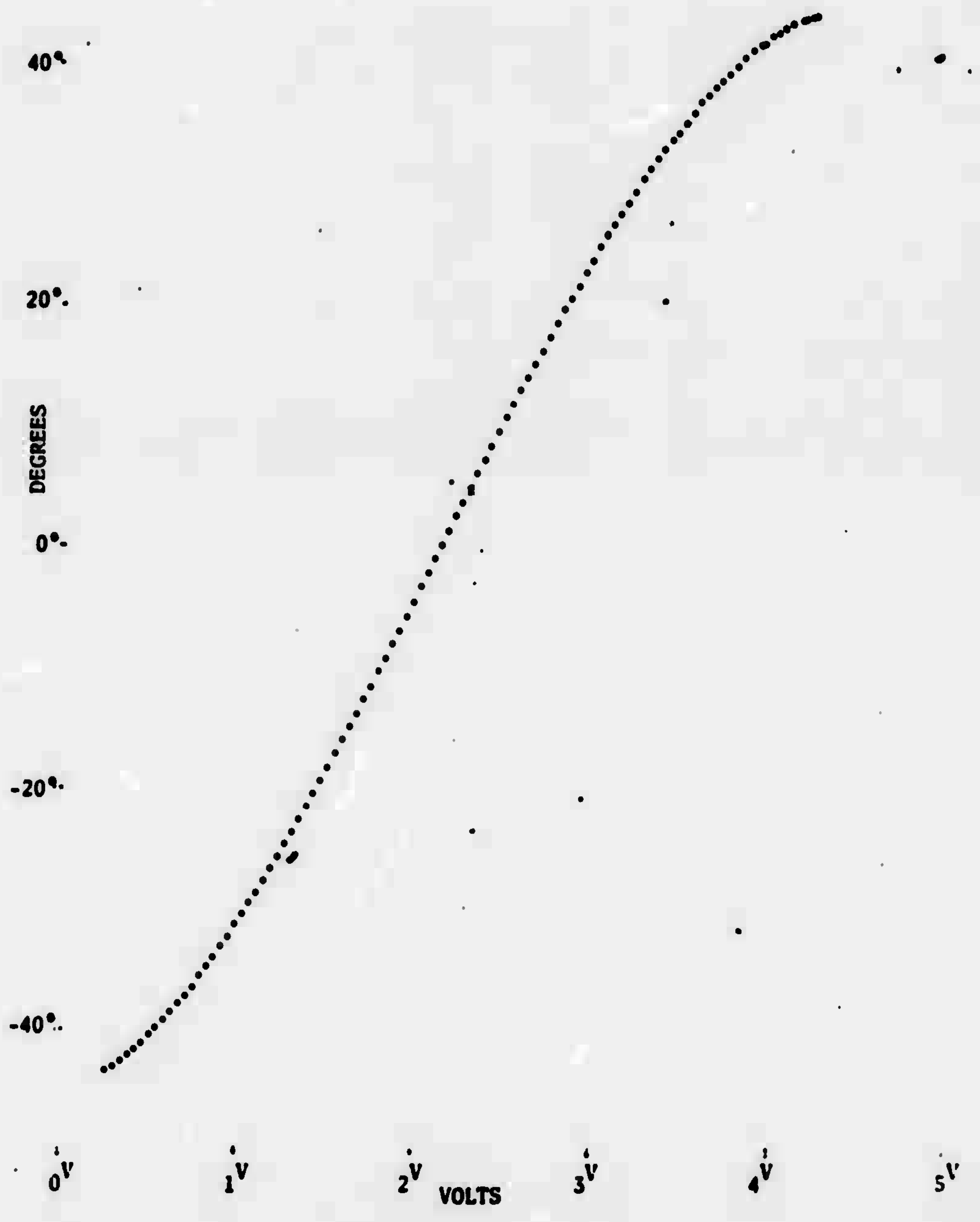


Figure 31

LEAST SQUARES APPROXIMATION FOR SUN SENSOR C

OUTPUT VOLTAGE 1

$\theta=-45.877+2.570V+12.816V^2-1.994V^3$

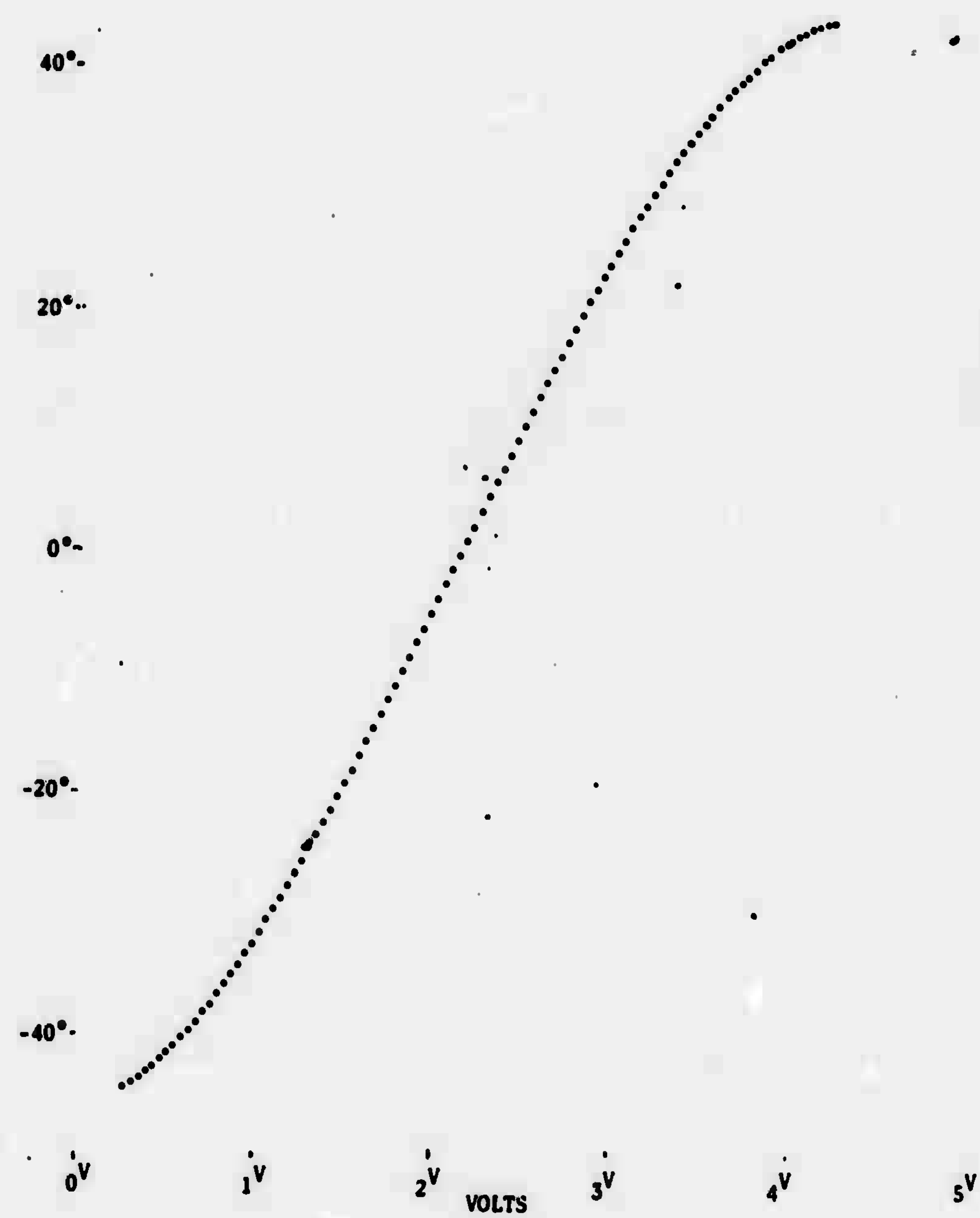


Figure 32

LEAST SQUARES APPROXIMATION FOR SUN SENSOR C

OUTPUT VOLTAGE 2

$$\theta=-46.958+4.022V+12.134V^2-1.9072V^3$$

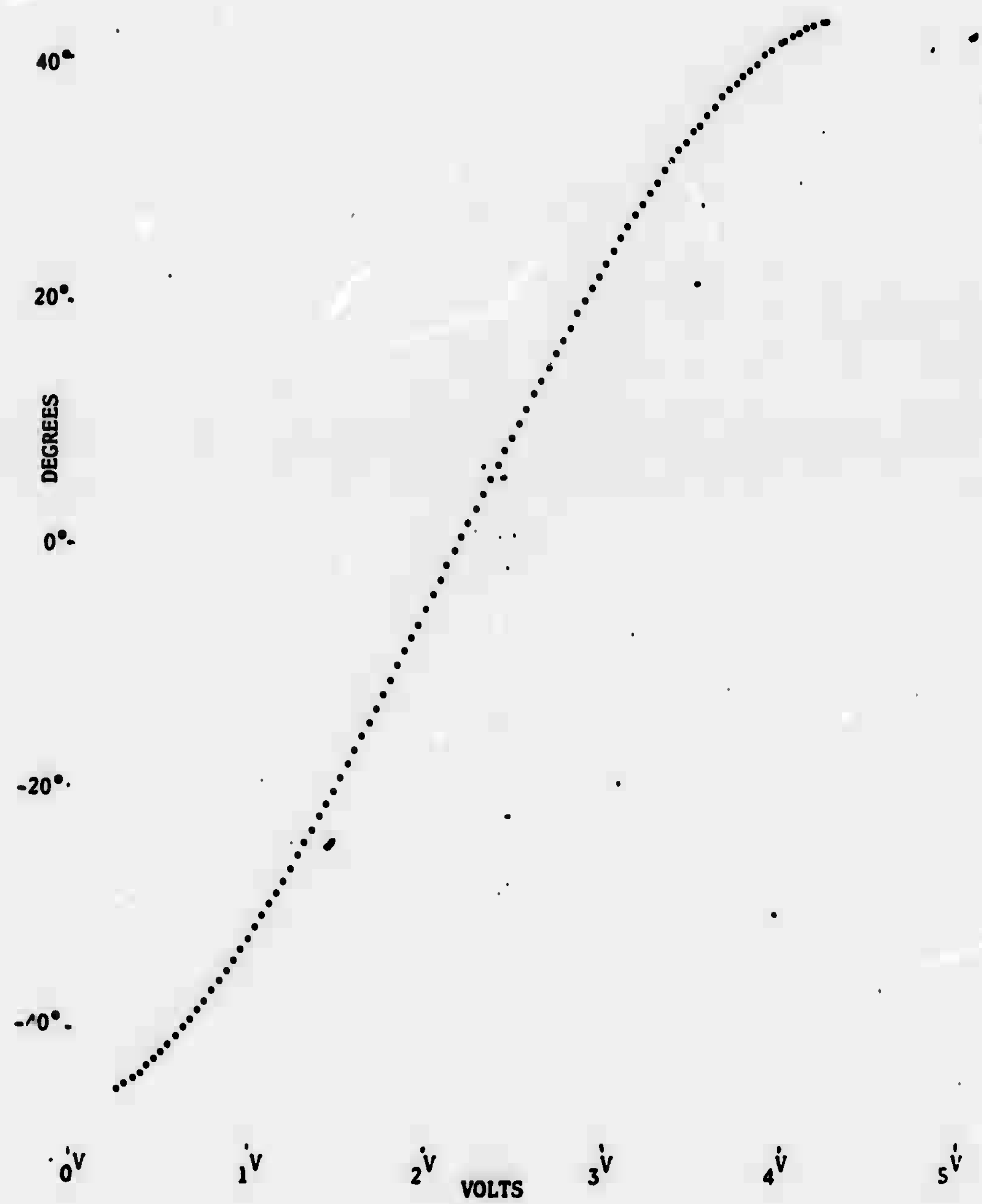


Figure 33

LEAST SQUARES APPROXIMATION FOR SUN SENSOR D

OUTPUT VOLTAGE 1

$$\Theta=-45.795+3.296V+12.528V^2-1.970V^3$$

Figure 34

LEAST SQUARES APPROXIMATION FOR SUN SENSOR D

OUTPUT VOLTAGE 2

$$\Theta=-45.594+3.668V+12.100V^2-1.885V^3$$

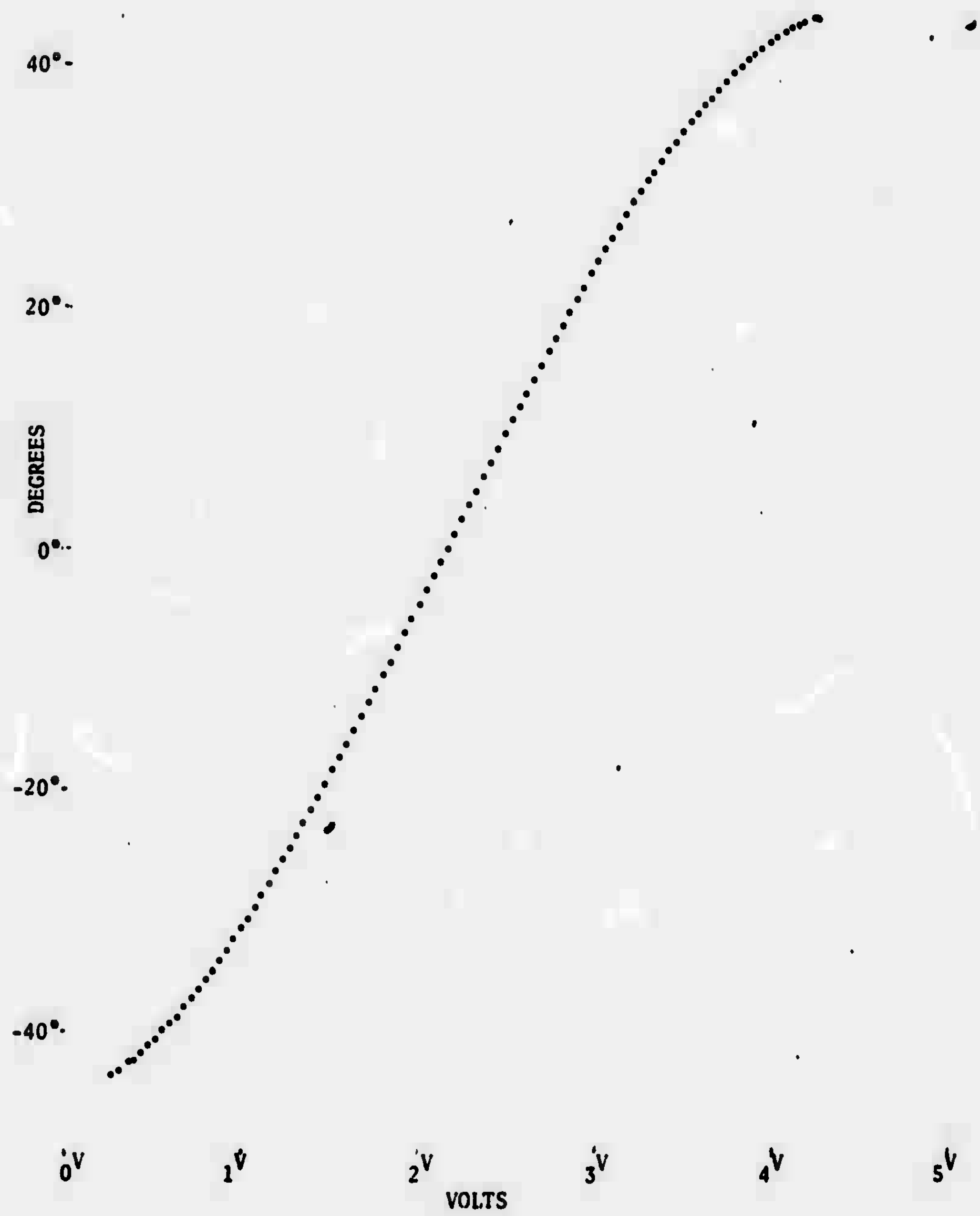


Figure 35

LEAST SQUARES APPROXIMATION FOR SUN SENSOR E

OUTPUT VOLTAGE 1

$\theta=-48.057+14.867V+5.154V^2-.851V^3$

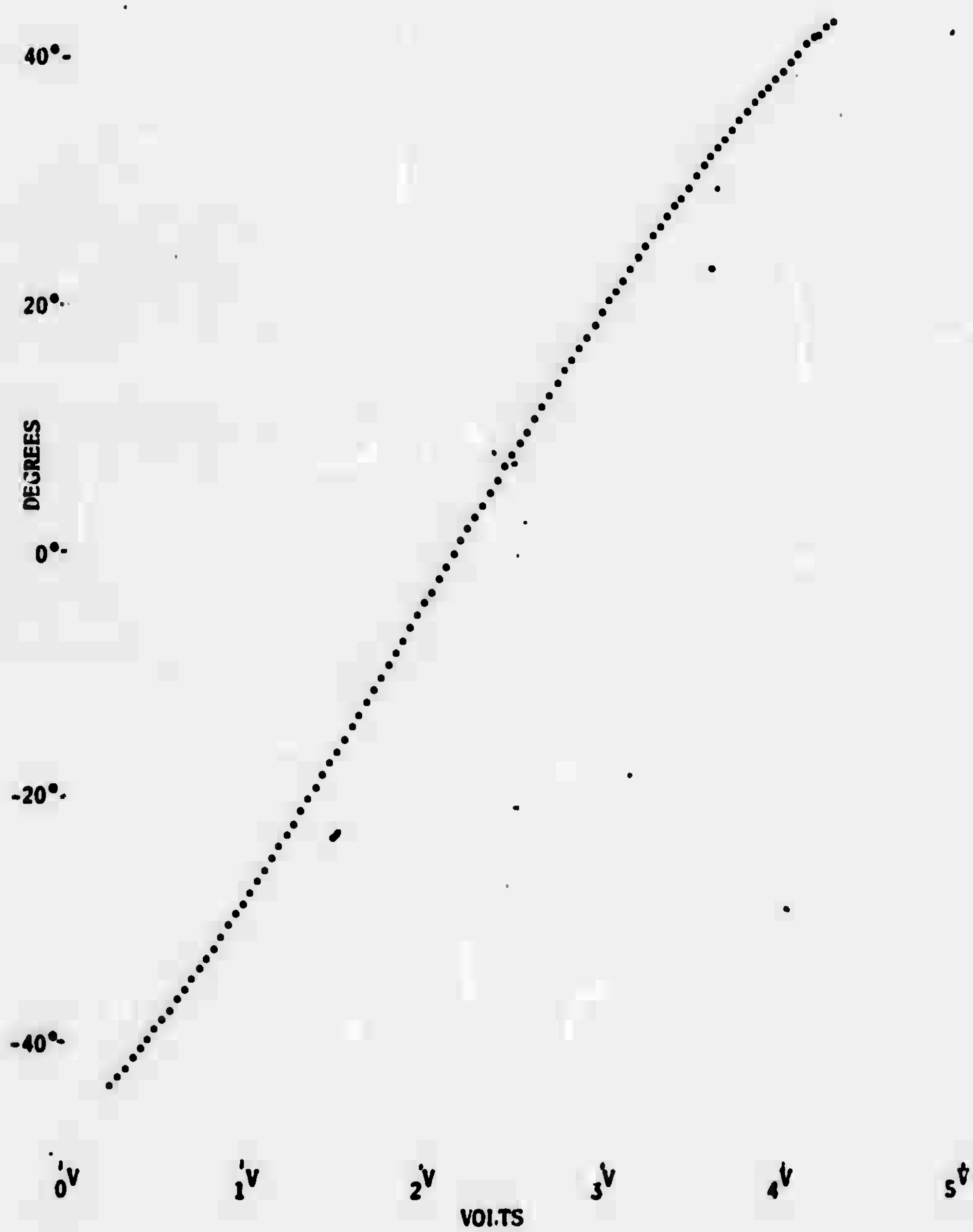


Figure 36

LEAST SQUARES APPROXIMATION FOR SUN SENSOR E

OUTPUT VOLTAGE 2

$\theta = +46.387 - 7.424V - 9.191V^2 + 1.400V^3$

Figure 37

LEAST SQUARES APPROXIMATION FOR SUN SENSOR F

OUTPUT VOLTAGE 1

$\Theta=-45.601+2.523V+12.483V^2-1.919V^3$

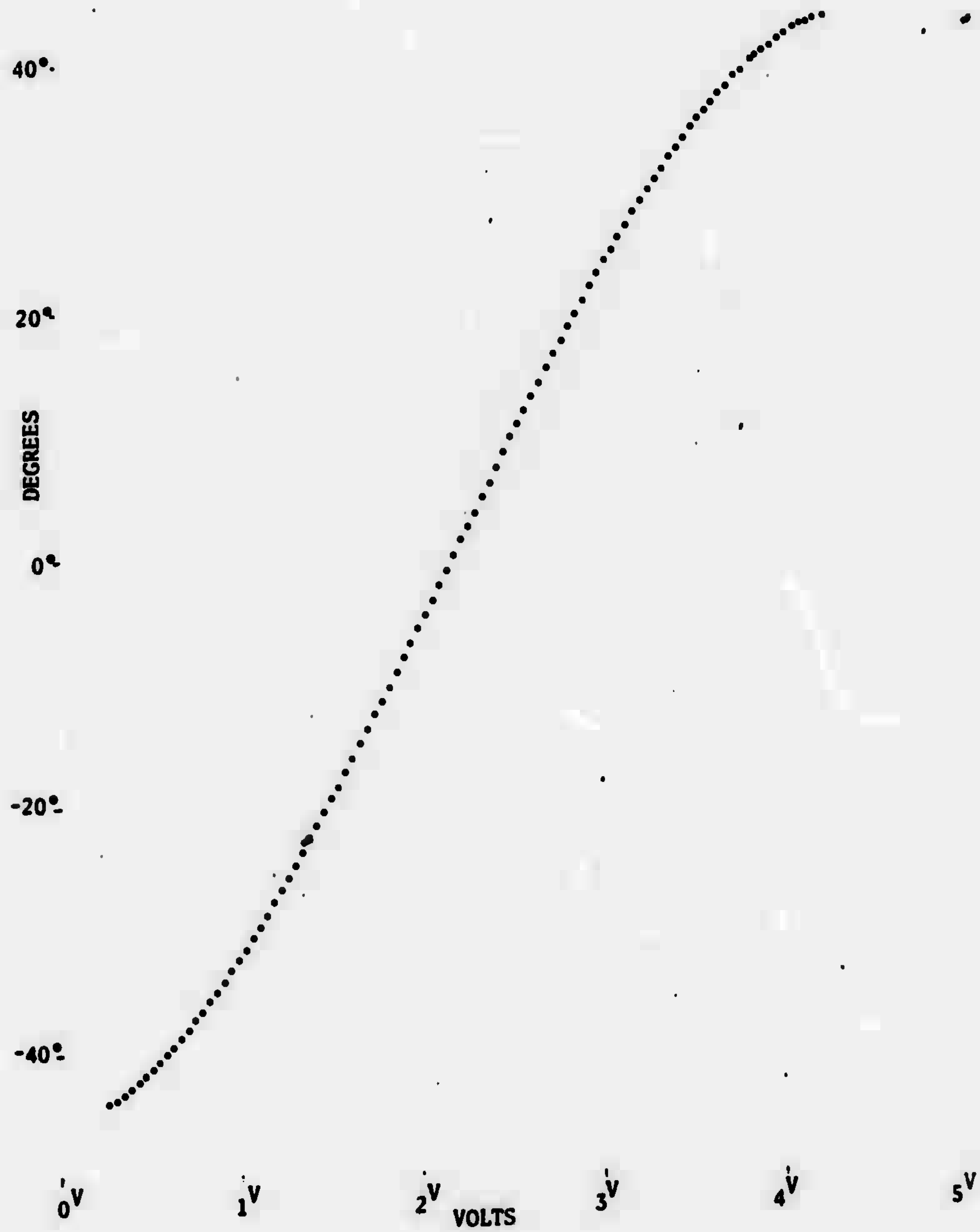
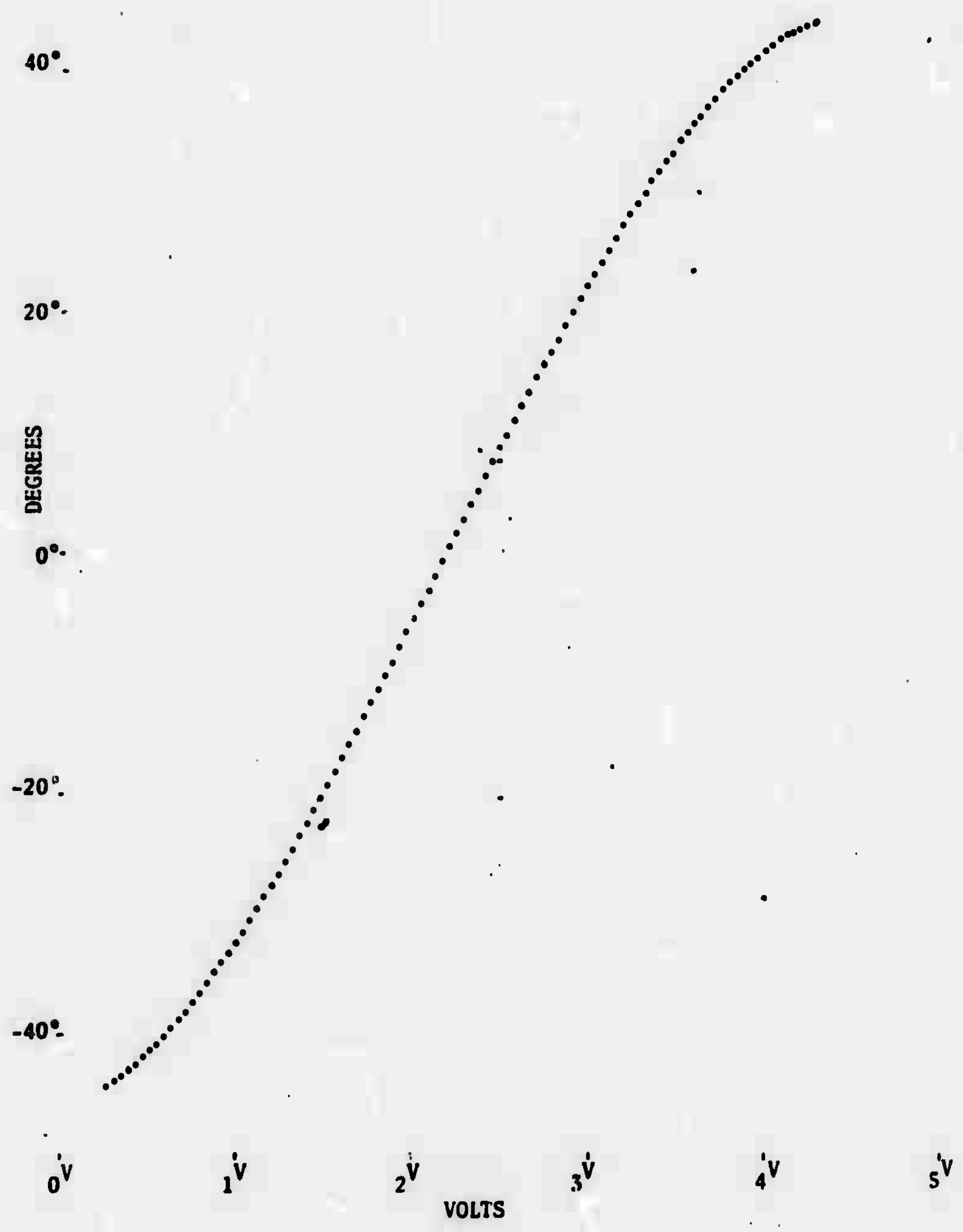


Figure 38

LEAST SQUARES APPROXIMATION FOR SUN SENSOR F

OUTPUT VOLTAGE 2

$$\Theta = -46.021 + 3.228V + 12.277V^2 - 1.895V^3$$



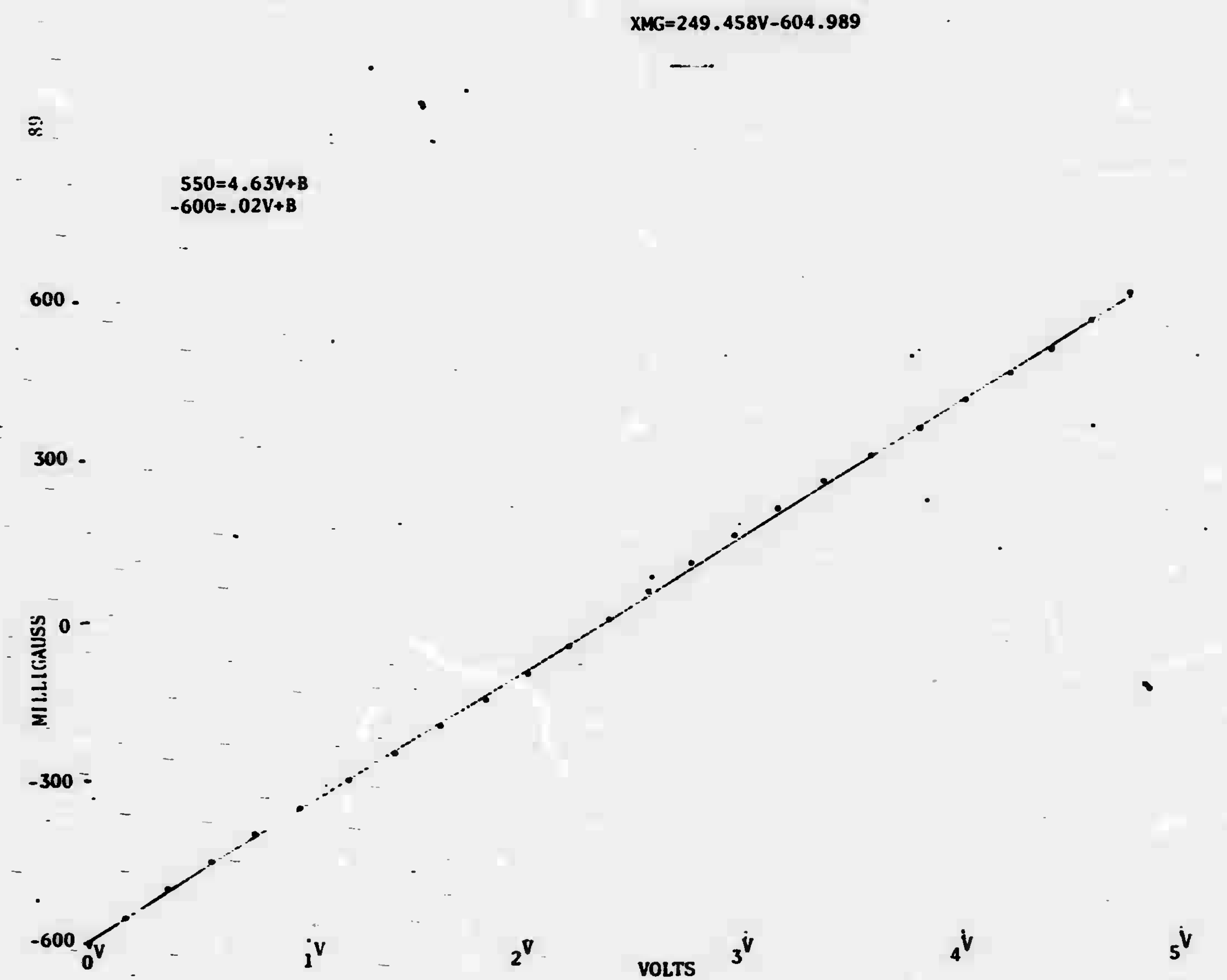

XMG=249.458V-604.989
550=4.63V+B
-600=.02V+B
600
300
0
-300
-600
MILLIGAUSS
0V
1V
2V
3V
4V
5V
VOLTS

Figure 39

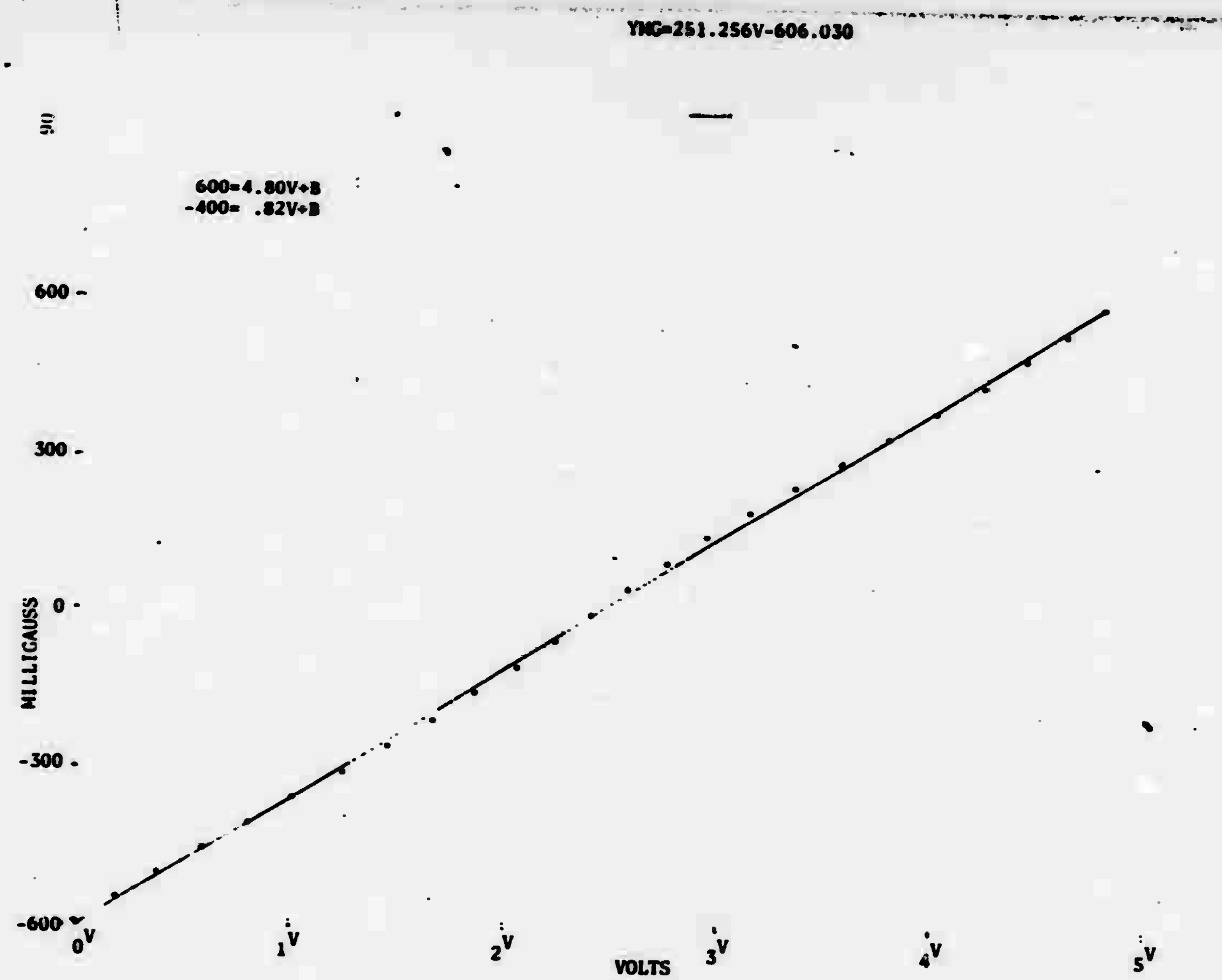

YMG=251.256V-606.030
600=4.80V+B
-400= .82V+B
600
300
0
-300
-600
MILLIGAUSS
0V
1V
2V
3V
4V
5V
VOLTS

Figure 40

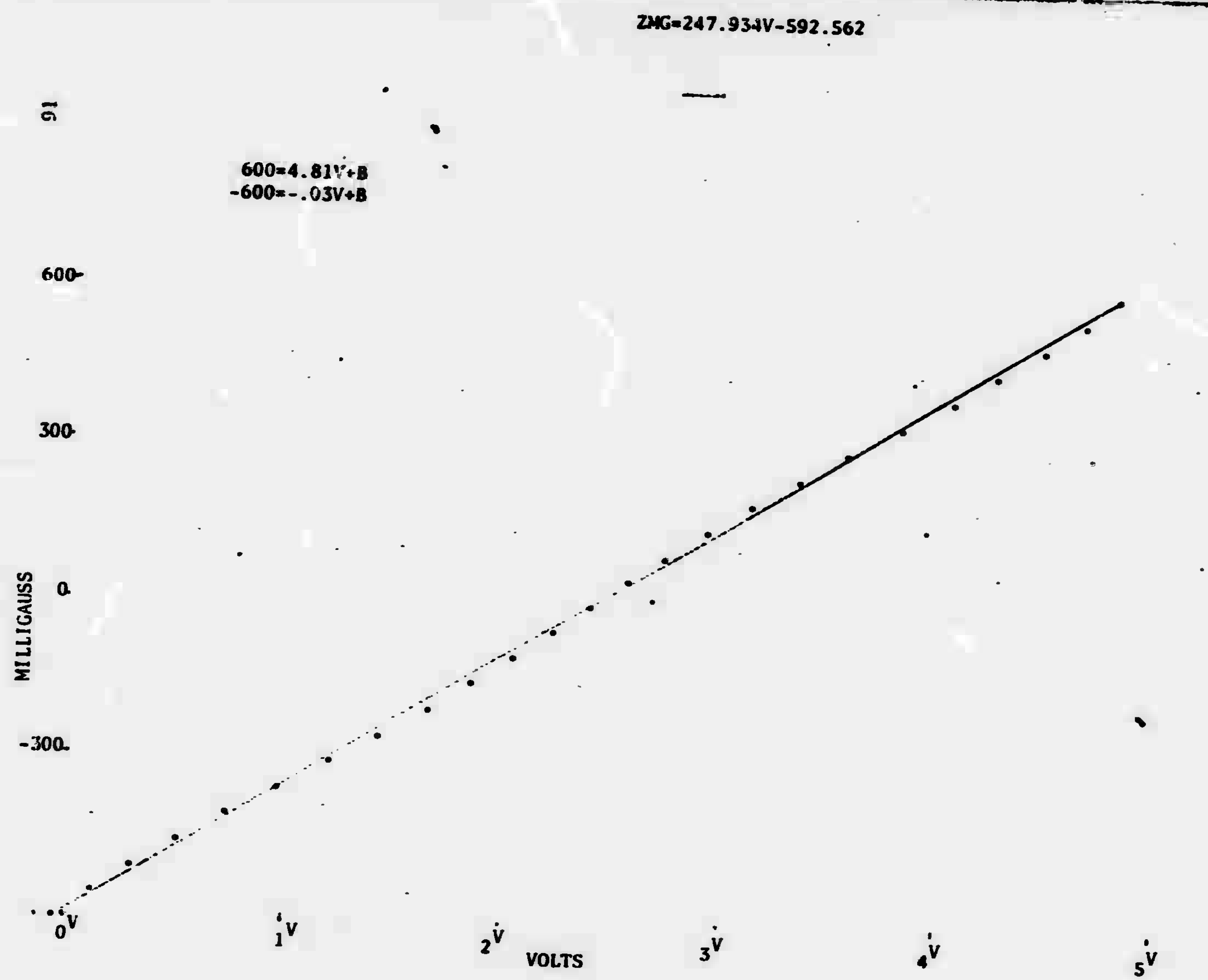

ZMG=247.934V-592.562
600=4.81V+B
-600=-.03V+B
600
300
0
-300
MILLIGAUSS
0V
1V
2V
3V
4V
5V
VOLTS

Figure 41

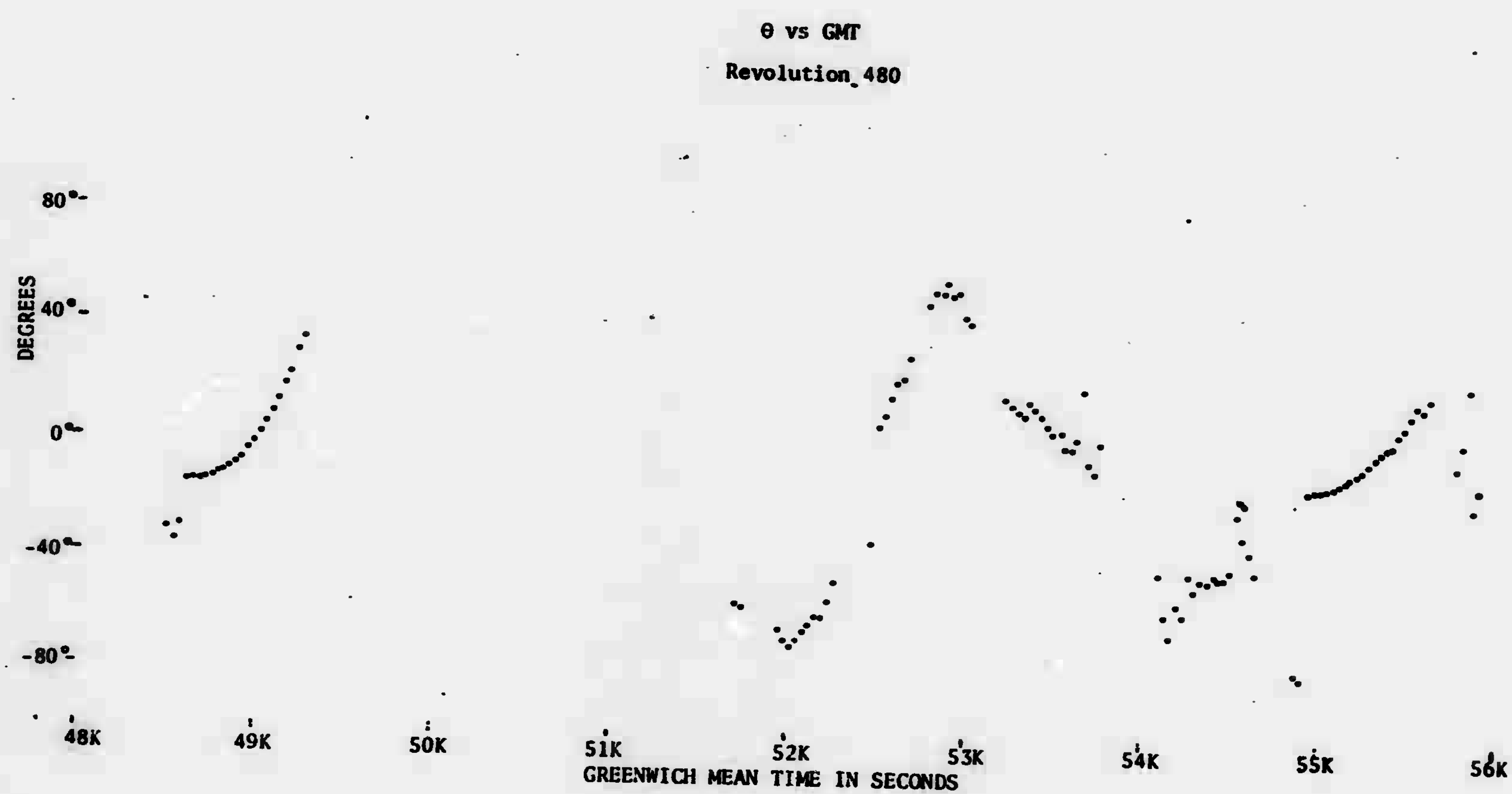

θ vs GMT
Revolution 480
80°
40°
0°
-40°
-80°
DEGREES
48K
49K
50K
51K
52K
53K
54K
55K
56K
GREENWICH MEAN TIME IN SECONDS

Figure 42

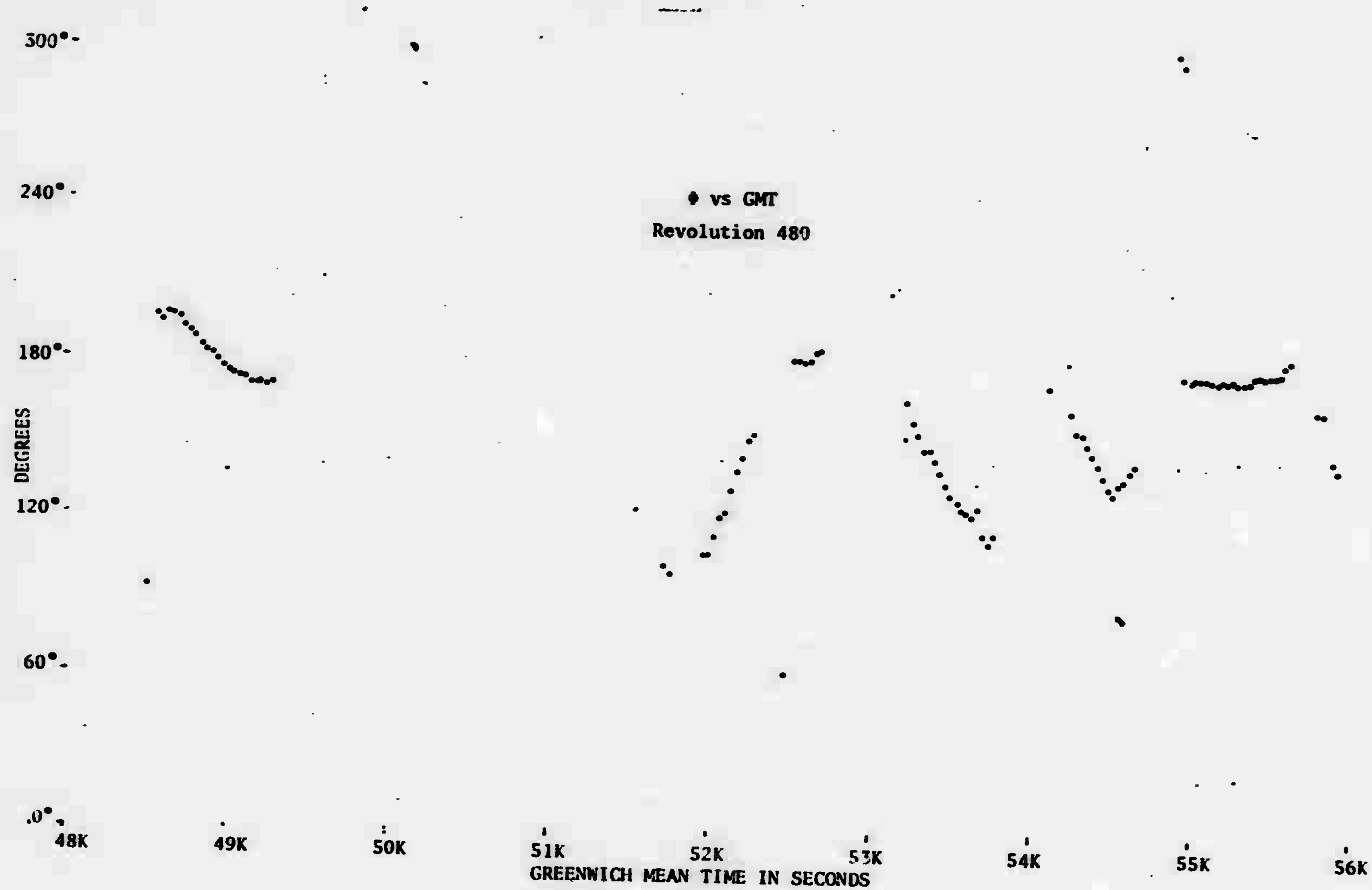

Φ vs GMT
Revolution 480
DEGREES
300°
240°
180°
120°
60°
0°
48K
49K
50K
51K
52K
53K
54K
55K
56K
GREENWICH MEAN TIME IN SECONDS

Figure 43

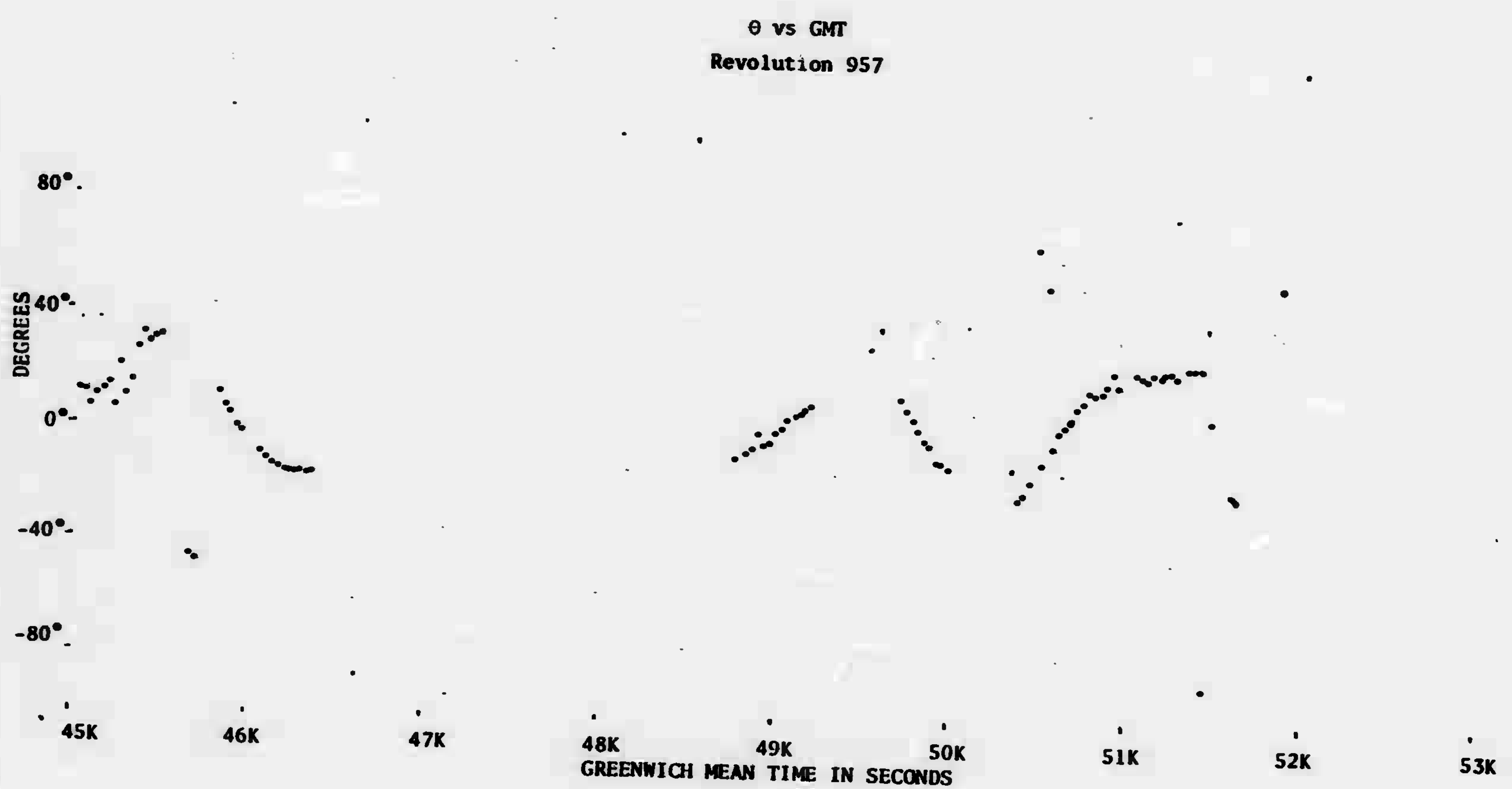

θ vs GMT
Revolution 957
DEGREES
80°
40°
0°
-40°
-80°
45K
46K
47K
48K
49K
50K
51K
52K
53K
GREENWICH MEAN TIME IN SECONDS

Figure 44

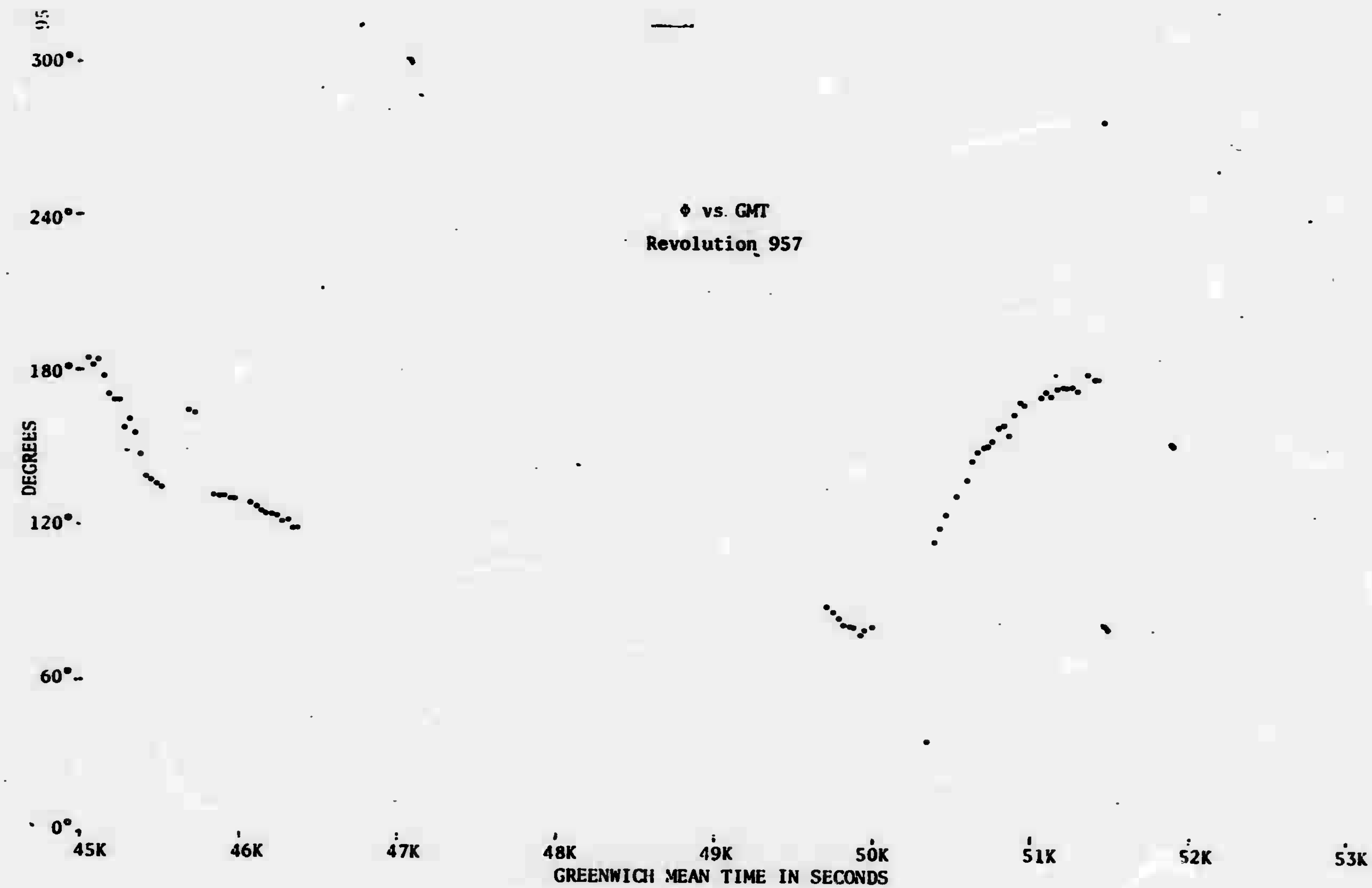

Φ vs. GMT
Revolution 957
DEGREES
300°
240°
180°
120°
60°
0°
45K
46K
47K
48K
49K
50K
51K
52K
53K
GREENWICH MEAN TIME IN SECONDS

Figure 45

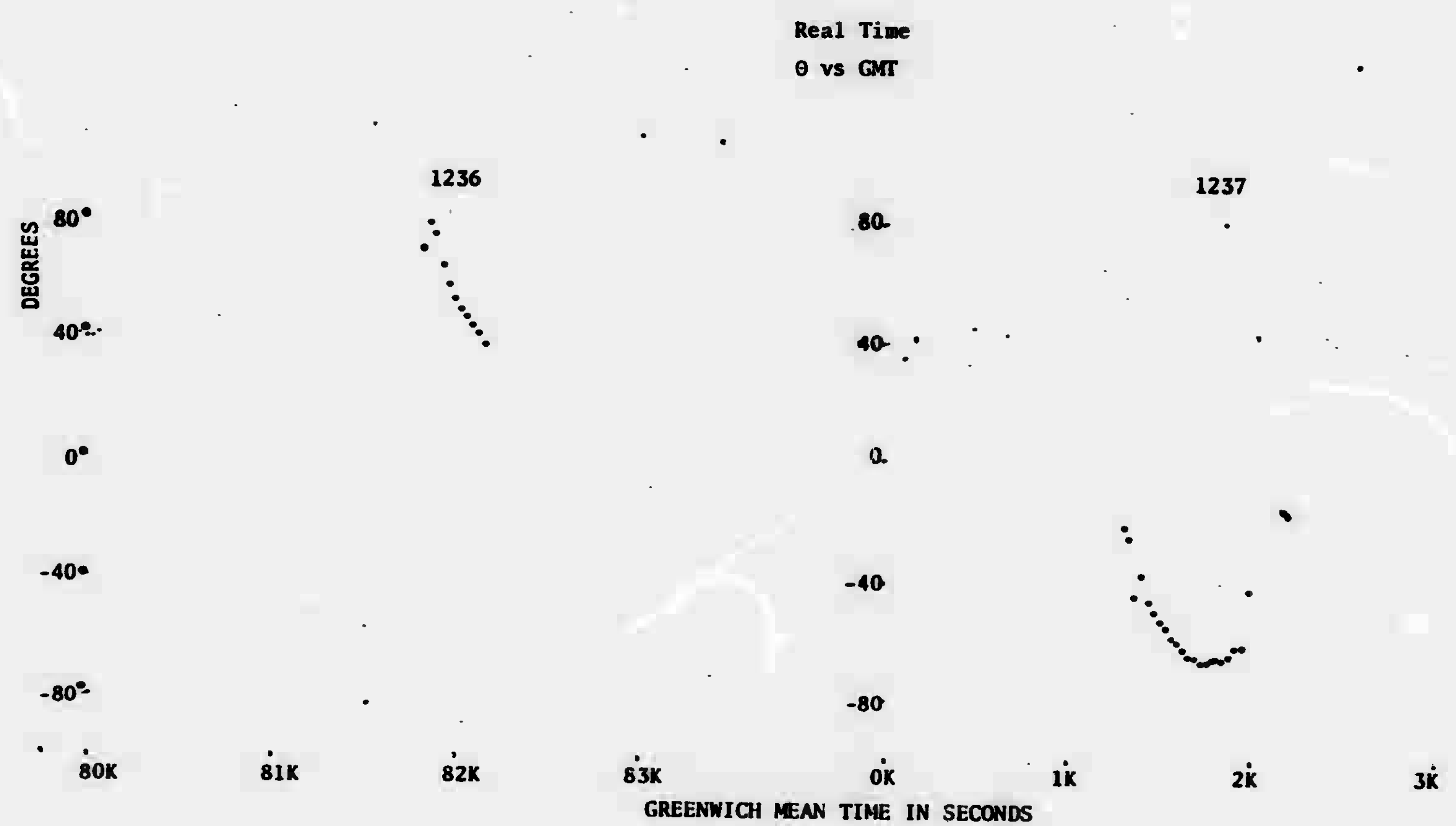

Real Time
θ vs GMT
DEGREES
80°
40°
0°
-40°
-80°
1236
1237
80
40
0
-40
-80
80K
81K
82K
83K
0K
1K
2K
3K
GREENWICH MEAN TIME IN SECONDS

Figure 46

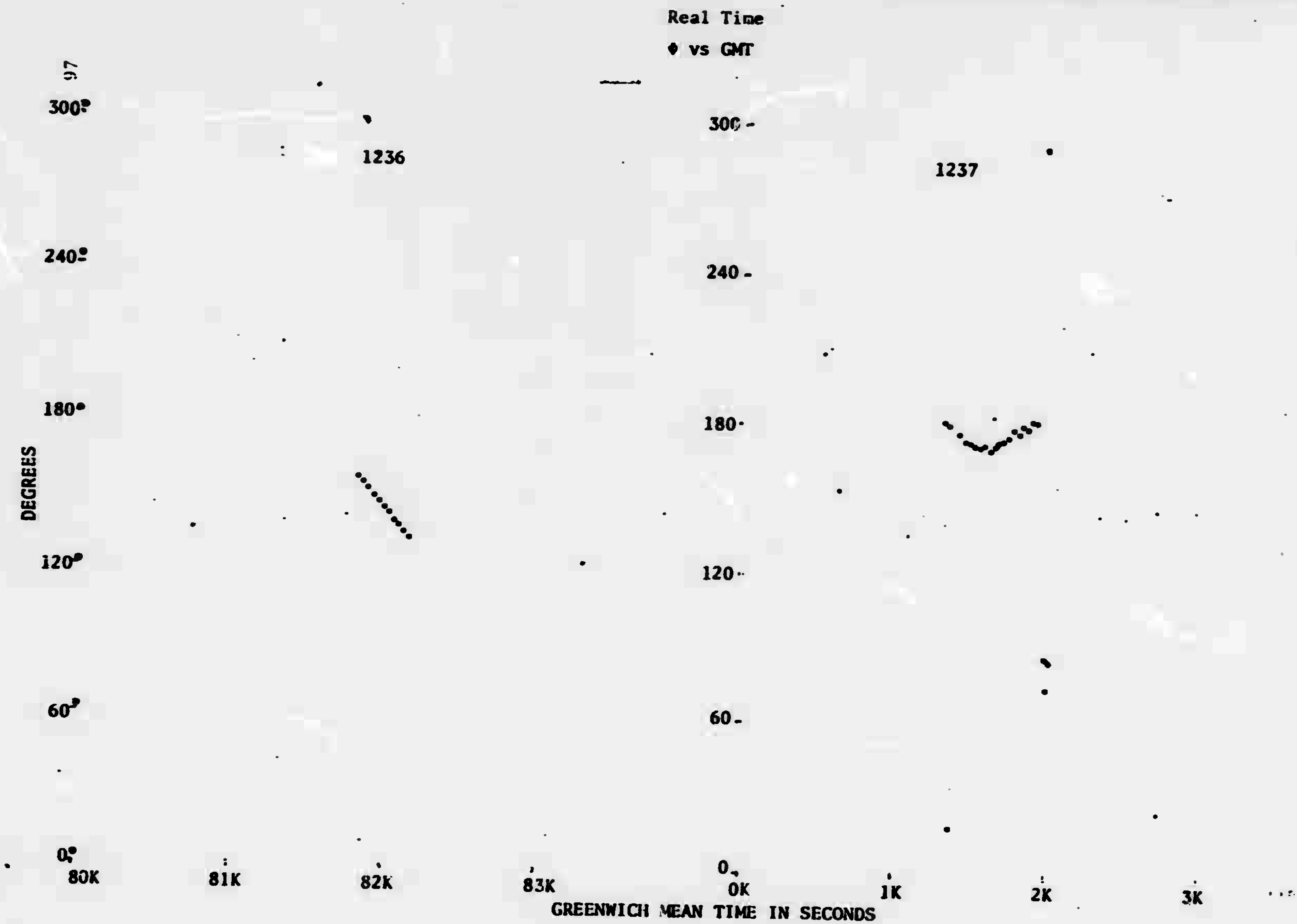

Real Time
φ vs GMT
300°
240°
180°
120°
60°
0°
DEGREES
1236
1237
300
240
180
120
60
0
80K
81K
82K
83K
0K
1K
2K
3K
GREENWICH MEAN TIME IN SECONDS

Figure 47

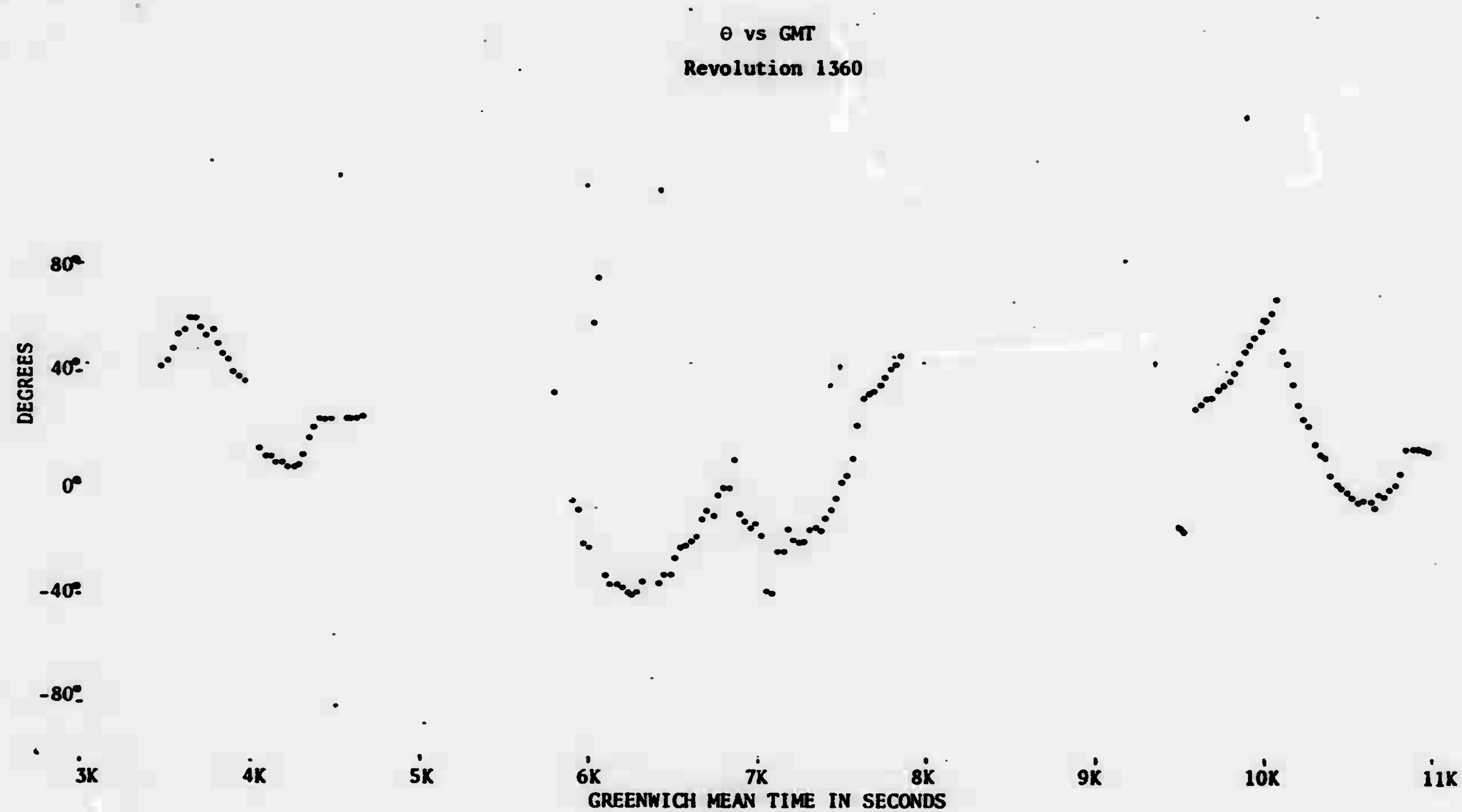

θ vs GMT
Revolution 1360
DEGREES
80°
40°
0°
-40°
-80°
3K
4K
5K
6K
7K
8K
9K
10K
11K
GREENWICH MEAN TIME IN SECONDS

Figure 48

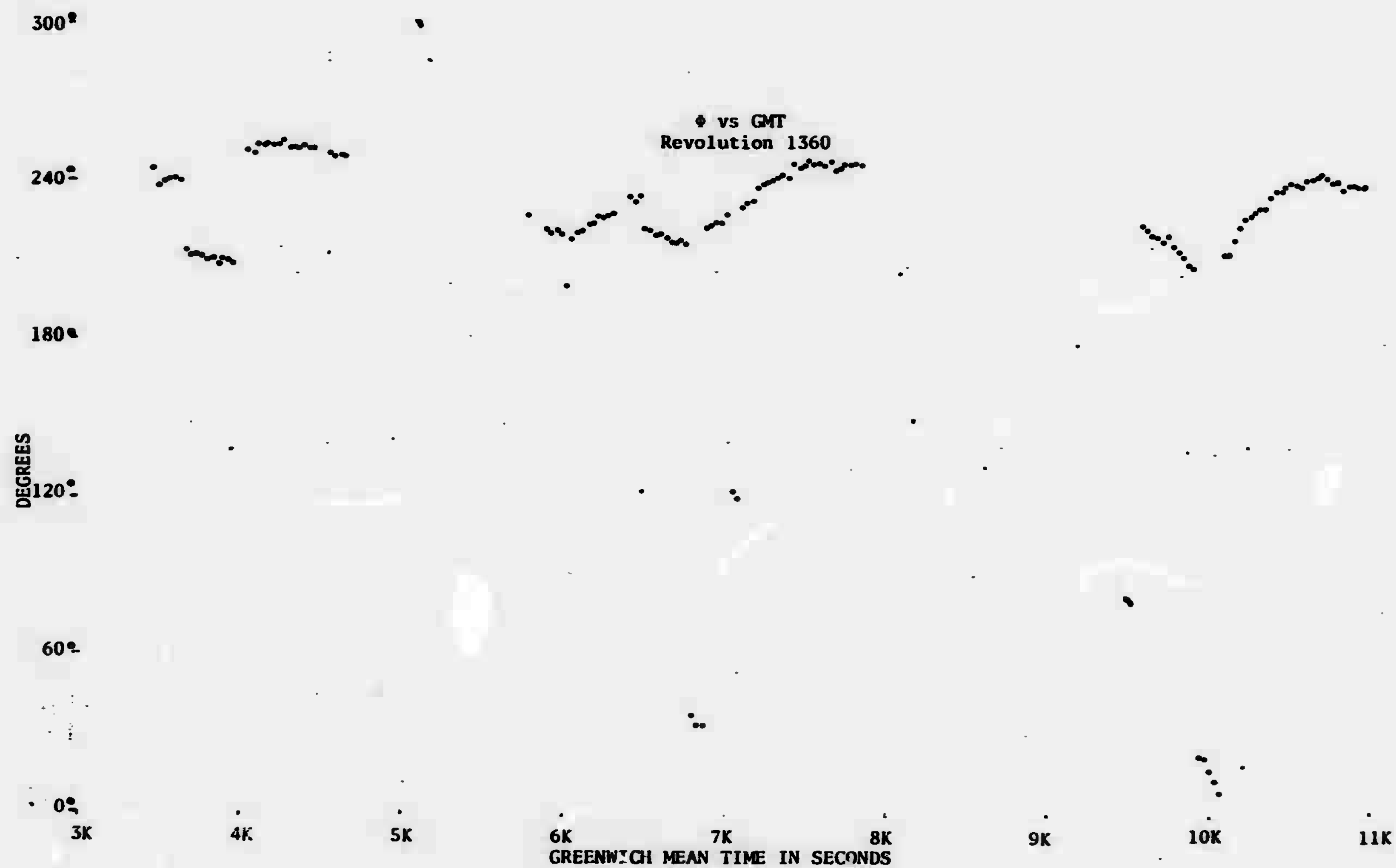

Φ vs GMT
Revolution 1360
DEGREES
300°
240°
180°
120°
60°
0°
3K
4K
5K
6K
7K
8K
9K
10K
11K
GREENWICH MEAN TIME IN SECONDS

Figure 49

APPENDIX F

In this section we will prove the equivalence of relations (66) and (69). It has been shown that

$$A_1 \cos\rho - A_2 \sin\rho = \cos\gamma_s \tag{72}$$

If we now make the substitution by letting

$$\tan\psi = \frac{A_2}{A_1}$$

then

$$\sin\psi = \frac{A_2}{\sqrt{A_1^2 + A_2^2}}$$
$$\cos\psi = \frac{A_1}{\sqrt{A_1^2 + A_2^2}} \tag{73}$$

Substituting (73) into (72) yields

$$\sqrt{A_1^2 + A_2^2}\,[\cos\psi \cos\rho - \sin\psi \sin\rho] = \cos\gamma_s$$

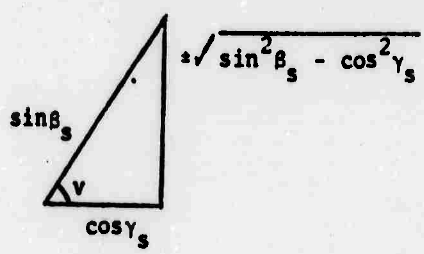
$$\sqrt{A_1^2 + A_2^2}\ \cos(\rho + \psi) = \cos\gamma_s$$

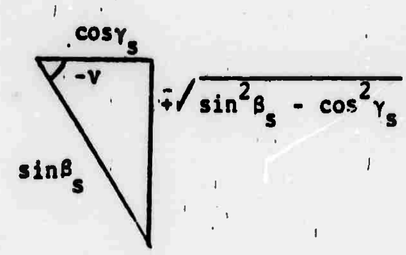
$$\cos(\rho + \psi) = \frac{\cos\gamma_s}{\sin\beta_s}$$

$$\Rightarrow \quad \rho = \pm \arccos\left(\frac{\cos\gamma_s}{\sin\beta_s}\right) - \psi \tag{74}$$

Let

$$\nu = \pm \arccos\left(\frac{\cos\gamma_s}{\sin\beta_s}\right)$$





It follows that

$$\cos(\pm v) = \frac{\cos\gamma_s}{\sin\beta_s}$$

$$\sin v = \frac{\pm\sqrt{\sin^2\beta_s - \cos^2\gamma_s}}{\sin\beta_s}$$

$$\sin(-v) = \frac{\mp\sqrt{\sin^2\beta_s - \cos^2\gamma_s}}{\sin\beta_s}$$

Let us restrict ourselves to

$$-v = \arccos\left(\frac{\cos\gamma_s}{\sin\beta_s}\right)$$

(It can be shown that if we take the positive sign for v, the analysis will result in an erroneous expression for (66)).

Therefore from (74)

$$\cos\rho = \frac{A_1\cos\gamma_s}{\sin^2\beta_s} - \frac{A_2(e_\phi e_r\hat{S})}{\sin^2\beta_s} \tag{75}$$

Multiplying and dividing by $\cos\Theta$,

$$\cos\rho = \frac{A_1\cos\gamma_s\cos\Theta + C(e_\phi e_r\hat{S})}{\sin^2\beta_s\cos\Theta} \tag{76}$$

However

$$A_1\cos\gamma_s\cos\Theta = \cos\gamma_s[\sin\Theta_s - \sin\Theta\cos\beta_s]$$

so the expression in (76) is in agreement with (66).

In a similar manner we find

$$\sin\rho = \frac{\mp A_1 \sqrt{\sin^2\beta_s - \cos^2\gamma_s}}{\sin^2\beta_s} - \frac{A_2 \cos\gamma_s}{\sin^2\beta_s}$$

or

$$\sin\rho = \frac{- A_1 (e_\phi e_r \hat{S}) - A_2 \cos\gamma_s}{\sin^2\beta_s} \tag{77}$$

The problem now is to show that $\sin\rho$ in (77) divided by $\cos\rho$ in (75) will yield the same expression as (69).

$$\tan\rho = \frac{\sin\rho}{\cos\rho} = \frac{-A_1(e_\phi e_r \hat{S}) - A_2 \cos\gamma_s}{A_1 \cos\gamma_s - A_2(e_\phi e_r S)}$$

Multiplying and dividing by the conjugate of the radical in the denominator we get

$$\tan\rho = \frac{[\mp A_1 \sqrt{\sin^2\beta_s - \cos^2\gamma_s} - A_2\cos\gamma_s]\,[A_1\cos\gamma_s \pm A_2 \sqrt{\sin^2\beta_s - \cos^2\gamma_s}]}{[A_1\cos\gamma_s \mp A_2\sqrt{\sin^2\beta_s - \cos^2\gamma_s}]\,[A_1\cos\gamma_s \pm A_2 \sqrt{\sin^2\beta_s \cos^2\gamma_s}]}$$

$$\tan\rho = \frac{-A_1A_2\cos^2\gamma_s \mp A_2^2\cos\gamma_s \sqrt{\sin^2\beta_s - \cos^2\gamma_s} \mp A_1^2\cos\gamma_s \sqrt{\sin^2\beta_s - \cos^2\gamma_s} - A_1A_2(\sin^2\beta_s - \cos^2\gamma_s)}{\cos^2\gamma_s(A_1^2 + A_2^2) - A_2^2 \sin^2\beta_s}$$

$$\tan\rho = \frac{\mp \cos\gamma_s \sqrt{\sin^2\beta_s - \cos^2\gamma_s}\;(A_1^2 + A_2^2) - A_1A_2 \sin^2\beta_s}{\sin^2\beta_s(\cos^2\gamma_s - A_2^2)}$$

$$\tan\rho = \frac{-A_1A_2 \mp \cos\gamma_s \sqrt{\sin^2\beta_s - \cos^2\gamma_s}}{\cos^2\gamma_s - A_2^2}$$

or

$$\tan\rho = \frac{A_1A_2 \pm \cos\gamma_s \sqrt{\sin^2\beta_s - \cos^2\gamma_s}}{A_2^2 - \cos^2\gamma_s}$$

which is in agreement with (69).